



Indiana State Math Contest
2023
Geometry/Integrated II
Exam

This test was prepared by faculty of
Franklin College,
Paul Fonstad, Ph.D.

Mark your calendar:

ICTM State Awards Ceremony 2023: Friday, June 9, 2023

ICTM State Math Contest 2024: Saturday, April TBA, 2024

**Do not open this test booklet until you have been
advised to do so by the test proctor.**

Indiana State Mathematics Contest

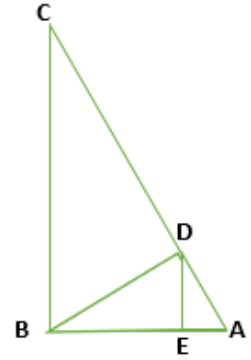
2023

Geometry

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1. The 30-60-90 triangle $\triangle CAB$ with right angle $\angle CBA$ has an altitude of \overline{BD} , and $\triangle BDA$ has an altitude of \overline{DE} . If the length of \overline{AE} is 1, what is the length of \overline{AC} ?



- a. 8
- b. 9
- c. $4\sqrt{3}$
- d. 6
- e. None of the above

2. When dilating a polygon, if its perimeter is tripled, what happens to its area?

- a. Multiplied by 4
- b. Multiplied by 9
- c. Multiplied by 3
- d. Multiplied by 6
- e. None of the above

3. Which of the following proper quadrilaterals has a rotational symmetry of 180° and 2 lines of symmetry? (A shape is *proper* if it cannot be classified more exactly, for example, a proper rectangle must have different measures for its length and its width.)

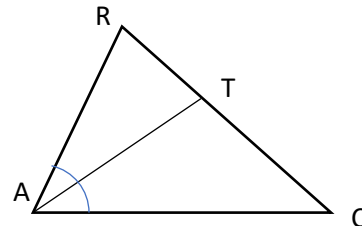
- a. A rhombus
- b. A square
- c. A parallelogram
- d. A trapezoid
- e. A kite

4. Consider the statement: "If the perimeter of a rectangle is 20, then the area is 24." Which of the following statements would be a counterexample to this statement?

- a. A rectangle with sides of length 3 and 8
- b. A rectangle with side of length 6 and 4
- c. A rectangle with sides of length 2 and 10
- d. A non-square rhombus with side length 5
- e. None of the above

5. If $\angle CAT \cong \angle RAT$, and if $m(\overline{AR}) = 15$, $m(\overline{AC}) = 20$ and $m(\overline{CR}) = 28$, find $m(\overline{CT})$.

- a. 16
- b. 12
- c. 20
- d. 8
- e. None of the above

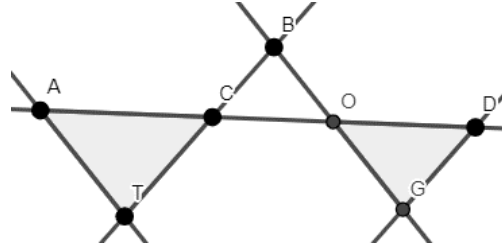


6. Find the center of the circle passing through the points (0,0), (0,6), and (8,0).

- a. (5,12)
- b. (4,3)
- c. (5,5)
- d. $(3\sqrt{2}, 2\sqrt{2})$
- e. None of the above

7. If $\overrightarrow{AT} \parallel \overrightarrow{BO}$ and $\overrightarrow{BC} \parallel \overrightarrow{DG}$, then

- a. $\triangle CAT$ is similar to $\triangle GOD$
- b. $\triangle CAT$ is similar to $\triangle DOG$
- c. $\triangle ACT$ is similar to $\triangle GOD$
- d. $\triangle ACT$ is similar to $\triangle DOG$
- e. None of the above

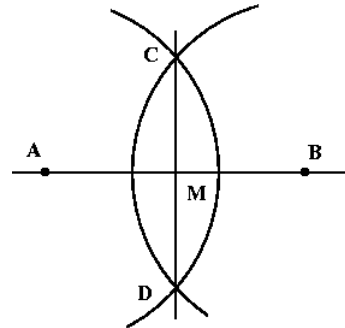


8. Let S be a square with side length 10 cm. If R is a rectangle with twice the perimeter and half the area of S , what is the length of the longer side of R to the nearest tenth of a centimeter.

- a. 17.1 cm
- b. 32.8 cm
- c. 36.4 cm
- d. 38.7 cm
- e. None of the above

9. Consider the geometric construction to the right, drawn using a non-collapsing compass. Which of the following statements is accurate?

- I. M is the midpoint of \overline{AB}
- II. $ACBD$ is a rhombus
- III. $2m(\angle CAM) = m(\angle CAD)$
- IV. $\angle CMB$ is a right angle



- a. I is false
- b. II is false
- c. III is false
- d. IV is false
- e. I-IV are all true

10. A regular dodecagon $ABCDEFGHIJKL$ is inscribed inside a circle P . Find the measure of $\angle HAG$.

- a. 30°
- b. 6°
- c. 12°
- d. 15°
- e. None of the above

11. Let $x, y,$ and z be the side lengths of a scalene triangle. Which of the following statements must be true?

- I. $xy > z$
- II. $x - y < z$
- III. $GCD(x, y, z) = 1$

- a. I
- b. II
- c. I and II
- d. II and III
- e. I, II and III

12. Larkin wants to know how tall the Soldiers and Sailors Monument is in Monument Circle. She notices that when she is standing 98 feet away, she has to look up at a 71° angle to see the top of the monument. IF her eye level is 4ft above the ground, to the nearest foot, how tall is the Soldiers and Sailors Monument?

- a. 93 feet
- b. 337 feet
- c. 312 feet
- d. 169 feet
- e. None of the above

13. If the area of a square inscribed inside a circle inscribed in a square is x , what is the area of the larger square?

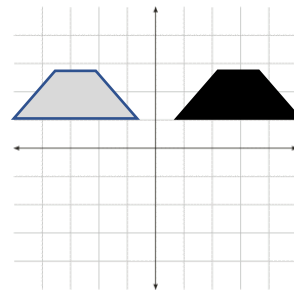
- a. $2\pi x$
- b. $x\sqrt{2}$
- c. πx^2
- d. $2x$
- e. None of the above

14. Let P be a regular n -sided polygon. If the interior angle measure of P is five times its exterior angle measure, how many sides does P have?

- a. 5
- b. 10
- c. 8
- d. 12
- e. None of the above

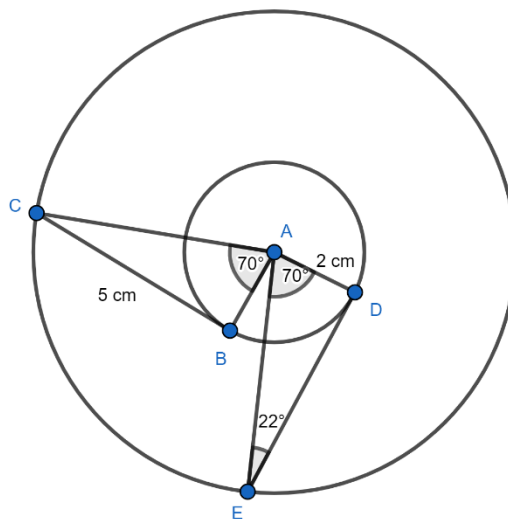
15. Which transformation of the plane would take the grey isosceles trapezoid to the black isosceles trapezoid in a single step?

- a. A reflection ONLY
- b. A translation ONLY
- c. A rotation ONLY
- d. A rotation or translation
- e. A reflection or translation

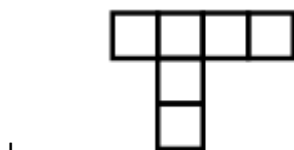


16. If the circles drawn are concentric circles both with center A , which justification could be used to show that these triangles are congruent?

- a. AAS
- b. SAS
- c. SSS
- d. Both SAS and AAS
- e. You would need more information to determine these triangles are congruent



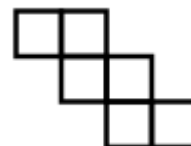
17. Which of the following is NOT a net for a cube?



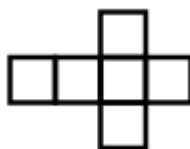
II.



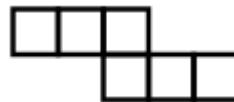
III.



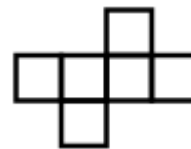
IV.



V.



VI.



- a. I and V
- b. I and II
- c. II and III
- d. III and VI
- e. II and V

18. If the length of the leg opposite angle θ in a right triangle is twice the length of the leg adjacent to θ , which of these values would be the greatest?

- a. $\sin \theta$
- b. $\tan \theta$
- c. $\cot \theta$
- d. 1
- e. Cannot be determined

19. Let $INTO$ be a given quadrilateral. Given that line segments \overline{IT} and \overline{NO} bisect each other, what is **the most** you can say about $INTO$?
- It is a quadrilateral.
 - It is a parallelogram.
 - It is a rhombus.
 - It is a rectangle.
 - It is a square.
20. Let A be a triangle. Consider the statement: "If A has a right angle, then A has exactly two acute angles." What is the inverse of this statement?
- If A has exactly two acute angles, then A has a right angle.
 - If A does not have exactly two acute angles, then A does not have a right angle.
 - If A does not have a right angle, then A does not have exactly two acute angles.
 - If A has a right angle, then A does not have exactly two acute angles.
 - None of the above.
21. In honor of Earth Day, GloboCorp decided to 'put a ring on it' by installing a circular metal band that went all the way around the Earth at the equator. Shortly after installation though, it was pointed out to them that the ring was blocking boat traffic and tripping people, making it less than ideal. To solve this issue, GloboCorp decided to expand the ring's circumference so that it could hover an even 1000 feet above the Earth's equator all the way around. To the nearest foot, by how much do they need to increase the ring's circumference to achieve this engineering feat?
- 31,415 feet
 - 1,918,862 feet
 - 271,828 feet
 - 6,283 feet
 - None of the above
22. Consider the line $Ax + By = C$, for given real numbers A, B, C . Which of the following lines would be perpendicular to it?
- $\frac{x}{B} - \frac{y}{A} = C$
 - $Ax - By = C$
 - $\frac{x}{A} - \frac{y}{B} = C$
 - $Bx + Ay = C$
 - None of the above

23. If point $A = (37, -13)$ and point $B = (43, -21)$, what is the distance from A to the midpoint of \overline{AB} ?

- a. $2\sqrt{5}$
- b. $5\sqrt{2}$
- c. 7
- d. 5
- e. None of the above

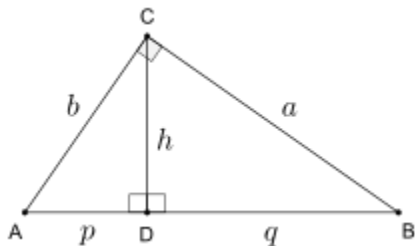
24. If a regular hexagon has a side length of 3.1 inches, how long would the side lengths of a square need to be for it to have the same area (to the nearest tenth of an inch)?

- a. 6.2 inches
- b. 5.3 inches
- c. 5.0 inches
- d. 5.7 inches
- e. None of the above

25. The straight cylindrical tubes that make up the AquaDuck water coaster on the Disney Fantasy are made of clear plastic. Each section of the tube is 96 inches long with an interior diameter of 48 inches and a tube thickness of 4 inches. How much clear plastic (in cubic inches) is needed to make each section of the tube to the nearest thousand cubic inches?

- a. $121,000 \text{ in}^3$
- b. $30,000 \text{ in}^3$
- c. $63,000 \text{ in}^3$
- d. $174,000 \text{ in}^3$
- e. None of the above

26. For the triangle below, if $p=5$ and $q=20$, find the length of a to the nearest tenth.



- a. 22.4
- b. 11.8
- c. 20.2
- d. 10.0
- e. None of the above

27. The frustum of a cone is the part of a cone that is left behind when the top of the cone is cut off by a plane that is parallel to the base. If a cone with a height 10 inches with a base radius of 10 inches is cut by a plane parallel to the base 5 inches above it, what is the lateral surface area of the resulting frustum to the nearest square inch? (Do not include the circular bases.)
- 111 in²
 - 333 in²
 - 569 in²
 - 351 in²
 - None of the above
28. If a sector of a circle has an arc whose length is equal to that circle's diameter, what is the degree measure of the arc to the nearest tenth?
- 229.2°
 - 114.6°
 - 171.9°
 - 98.6°
 - None of the above
29. If \overleftrightarrow{AB} and \overleftrightarrow{CD} are parallel lines and M is a point in the plane, what would you need to know to be able to prove that $\triangle AMB$ is congruent to $\triangle CMD$?
- $\angle AMB \cong \angle CMD$.
 - M is the midpoint of \overline{AC} .
 - M is the midpoint of \overline{AD} .
 - $m(\overline{AB}) = m(\overline{CD})$.
 - None of the above are sufficient on their own.
30. Which platonic solid has 20 vertices and 30 edges?
- An octahedron
 - A cube
 - A dodecahedron
 - An icosahedron
 - A tetrahedron
31. Consider the points (1,0) and (0,1) in the plane. Which of the following given pairs of points could be used with the given points to make a square?
- (0,0) and (1,1).
 - (1,2) and (2,1).
 - (-1,0) and (0,-1).
- Only I could make a square
 - Only II could make a square
 - Only III could make a square
 - Exactly two of these could make a square.
 - All of these could make a square.

32. To determine the height of a giant, a little girl named Sophie, whose eyes are 4ft above the ground, places a mirror on the ground exactly 15 feet away from the giant. Walking backward, Sophie discovers that if she stands exactly 2 feet 6 inches behind the mirror, she can see the top of the giant's head. To the nearest inch, how tall is the giant?
- a. 20 feet 6 inches
 - b. 23 feet 0 inches
 - c. 22 feet 3 inches
 - d. 24 feet 0 inches
 - e. None of the above