



# Indiana State Math Contest

**2023**

**Algebra II/Integrated III**

**Exam**

This test was prepared by faculty of  
University of Southern Indiana

Mark your calendar:

*ICTM State Awards Ceremony 2023: Friday, June 9, 2023*

*ICTM State Math Contest 2024: Saturday, April TBA, 2024*

Do not open this test booklet until you have been  
advised to do so by the test proctor.

1. Simplify the expression  $\frac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}}$

- a.  $xy$       b.  $\frac{1}{xy}$       c.  $x + y$       d.  $\frac{xy}{x+y}$       e.  $\frac{y-x}{xy}$
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2. Simplify the expression  $\sqrt[12]{x^{15}} \cdot \frac{\sqrt[16]{x^{12}}}{\sqrt[15]{x^9}}$

- a.  $x^2\sqrt[7]{x}$       b.  $x\sqrt[5]{x^2}$       c.  $x^2\sqrt[5]{x^3}$       d.  $\sqrt[15]{x^7}$       e.  $\sqrt[7]{x^5}$
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3. Simplify the expression  $\frac{(x^{-2}y^3z)^{-4}}{x^3y^{-3}z^2}$

- a.  $\frac{x^5}{y^9z^6}$       b.  $\frac{x^9y^2}{z^6}$       c.  $\frac{x^7y^6}{z^2}$       d.  $\frac{x^{20}z^4}{y^{24}}$       e.  $\frac{z^4}{x^{20}y^{24}}$
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4. If  $y = f(x) = \frac{x+2}{x-1}$ , then it is **incorrect** to say

- a.  $x = \frac{y+2}{y-1}$       b.  $f(0) = -2$       c.  $f^{-1}(x) = \frac{x-1}{x-2}$       d.  $f(-2) = 0$       e.  $f(y) = x$
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5. Let  $f(x) = x^2$  and  $g(x) = x^2 + 5$  Find  $(f \circ g)(-3)$ .

- a. -16      b. 16      c. 86      d. 126      e. 196
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6. Which of the following is an extraneous solution of  $\frac{3}{x+2} + \frac{2}{x} = \frac{4x-4}{x^2-4}$

- a. -2      b. 0      c. 2      d. 4      e. None of these
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7. If  $f(x) = 2x - 3$ , then  $f^{-1}(x) =$

- a.  $\frac{1}{2x+3}$       b.  $\frac{1}{2}x + 3$       c.  $\frac{x+3}{2}$       d.  $\frac{3-x}{2}$       e.  $3 - 2x$
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8. Assuming  $f^{-1}(x)$  exists, if  $(3, -4)$  is in  $f(x)$  which ordered pair **must** be in  $f^{-1}(x)$ ?

- a.  $(-3, 4)$       b.  $(3, -4)$       c.  $(-4, 3)$       d.  $(\frac{1}{3}, -\frac{1}{4})$       e.  $(4, 3)$
- 

9. How many integer pairs  $(x, y)$  satisfy both  $|x + y| \leq 5$  and  $|x - y| \leq 5$ ?

- a. 25      b. 50      c. 61      d. 100      e. 101
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10. Describe the transformations performed on the graph of  $f(x) = \log_b x$  to graph  $g(x) = -3\log_b(x+2)$
- Reflection across the  $x$ -axis, vertically stretched by a factor of 3, and shifted left 2 units.
  - Reflection across the  $x$ -axis, vertically stretched by a factor of 3, and shifted right 2 units.
  - Reflection across the  $y$ -axis, vertically stretched by a factor of 3, and shifted left 2 units.
  - Reflection across the  $y$ -axis, vertically stretched by a factor of 3, and shifted right 2 units.
  - Reflection across the  $y$ -axis, vertically stretched by a factor of 3, and shifted up 2 units.
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11. A bag contains 6 red, 8 blue, and 10 yellow marbles. You grab a handful of 6 at random. Which number below is closest to the chance that you get 2 of each color?
- a.  $\frac{1}{3}$       b.  $\frac{1}{4}$       c.  $\frac{1}{5}$       d.  $\frac{1}{6}$       e.  $\frac{1}{7}$
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12. Which function models the value of a car that initially sells for \$30,000, has a salvage value of \$2,000, and loses 10% of its depreciable value each year?
- a.  $V(t) = -28000(1.1)^t + 2000$       b.  $V(t) = 28000(0.9)^t + 2000$   
c.  $V(t) = 28000(-0.1)^t + 2000$       d.  $V(t) = -30000(1.1)^t + 2000$   
e.  $V(t) = 30000(0.9)^t + 2000$
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13. Write  $2\log_b x - \log_b 3x + \log_b 5$  as a single logarithm

a.  $\log_b \frac{5x}{3}$       b.  $\log_b \frac{10x}{3}$       c.  $\log_b 15x^2$       d.  $\log_b(x^2 - 3x + 5)$       e.  $\log_b \frac{6x^2}{5}$

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14. What is the domain of  $f(x) = 4\log_b(3 - 2x)$ ?

a.  $(-\infty, \frac{3}{2})$       b.  $(\frac{3}{2}, \infty)$       c.  $(-\frac{3}{2}, \infty)$       d.  $(-\infty, -\frac{3}{2})$       e.  $(-\infty, \infty)$

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15. A single bacterium is placed in a container at 8:00 AM. If the number of cells doubles every minute and the container is filled at exactly 10:00 AM, at what time was the container  $\frac{1}{4}$  full?
- a. 8:30 AM      b. 8:48 AM      c. 9:00 AM      d. 9:30 AM      e. 9:58 AM
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16. Find the value of  $a + b + c$  in the following system  $\begin{cases} 2a - b + 5c = 15 \\ 2a + 3b + c = 75 \end{cases}$
- a. 26      b. 30      c. 48      d. 77      e. 87
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17. Find the sum of all  $x$  and  $y$  values that satisfy the following  $\begin{cases} \sqrt{x} + \sqrt{y} = 7 \\ 4\sqrt{x} - 3\sqrt{y} = 14 \end{cases}$
- a. 25      b. 29      c. 32      d. 45      e. 61
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18. Simplify  $(2 - 3i)^4$

- a.  $8 - 12i$       b.  $16 - 81i$       c.  $-119 + 120i$       d.  $8 + 12i$       e.  $16 + 81i$
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19. Find the discriminant of  $3x^2 + 2x = -1$  and describe the nature of the solutions

- a. Discriminant =  $-10$ , two distinct imaginary solutions  
b. Discriminant =  $-8$ , two distinct imaginary solutions  
c. Discriminant =  $0$ , one repeated real-number solutions  
d. Discriminant =  $4$ , two distinct real-number solutions  
e. Discriminant =  $16$ , two distinct real-number solutions
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20. Express the quadratic function  $f(x) = 4x^2 - 8x - 3$  in vertex form.

- a.  $f(x) = 4(x - 1)^2 - 7$       b.  $f(x) = 4(x - 1)^2 - 4$       c.  $f(x) = 4(x - 1)^2 + 1$   
d.  $f(x) = 4(x + 1)^2 - 7$       e.  $f(x) = 4(x + 1)^2 + 3$
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21. Find the zeros of  $f(x) = 2x^2 - 5x + 1$

- a.  $\frac{5 \pm \sqrt{17}}{4}$       b.  $\frac{5 \pm \sqrt{17}}{2}$       c.  $\frac{5 \pm \sqrt{-33}}{4}$       d.  $\frac{-5 \pm \sqrt{17}}{4}$       e.  $\frac{-5 \pm \sqrt{33}}{4}$
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22. What is the solution set of  $x^4 - 15x^2 - 16 = 0$  ?

- a.  $\{4, -4, 1, -1\}$       b.  $\{4, -4, i, -i\}$       c.  $\{4i, -4i, 1, -1\}$       d.  $\{16, 1\}$       e.  $\{-16, -1\}$
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23. What is the maximum value of  $f(x) = 1 + 2x - 2x^2$  ?

- a.  $\frac{1}{2}$       b.  $1\frac{1}{2}$       c.  $2$       d.  $-1$       e. No maximum exists
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24. How many solutions can the equation  $x^3 - 9x = k$  have?

- a. 0 or 2      b. 1 or 3      c. 1,2, or 3      d. 0,1,2, or 3      e. 1,3,5,7,or 9
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25. Alan, Bonnie, and Chad purchase lunch from the Food Court. Alan buys 1 taco and 2 fruit cups for \$5.25. Bonnie buys 3 tacos and 1 fruit cup for \$8.25. How much will Chad pay for 2 tacos and 2 fruit cups?

- a. \$6.00      b. \$6.50      c. \$7.00      d. \$7.50      e. \$8.00
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26. A model rocket is launched with an initial velocity of  $100 \frac{\text{ft}}{\text{sec}}$  from the top of a hill 20 feet high. Its height in feet  $t$  seconds after launch is given by the function  $h(t) = -16t^2 + 100t + 20$ .

Which of the following statements is **false** ?

- I. The rocket reaches a maximum height 3.125 seconds after launch
  - II. The rocket reaches a height of 120 feet 1.25 seconds after launch
  - III. The rocket reaches a height of 120 feet 5 seconds after launch
  - IV. The rocket hits the ground after 6.25 seconds
- a. I only      b. II only      c. III only      d. IV only      e. I-IV are all true

27. Bill has two children, Alex and Chris. Currently, Bill is 2 years younger than 4 times Alex's age. When Chris was Alex's age, Bill was 2 years older than 3 times Chris' age. Fifteen years from now, Alex and Chris' ages will total Bill's. How old was Bill when Alex was born?

- a. 28    b. 32    c. 35    d. 37    e. 41

28. The number of digits in  $2023^{2023}$  is

- a. 6688      b. 6689      c. 15399      d. 15400      e. More than  $10^{10}$

29. What is the remainder when  $x^3 - 3x^2 + 2$  is divided by  $x - 5$  ?

- a. -198      b. -98      c. 12      d. 52      e.  $-\frac{2}{5}$

30. The function  $f(x)$  is a polynomial. If  $f(x^2 + 1) = x^4 + 5x^2 + 3$ , what is  $f(x^2 - 1)$ ?

- a.  $x^4 - x^2 + 3$       b.  $x^2 + 3x + 1$       c.  $x^2 - 3x - 1$       d.  $x^4 + 5x^2 + 3$       e.  $x^4 + x^2 - 3$

31. Find the inverse of  $f(x) = \begin{cases} 4x & \text{if } x < 5, \\ 5/x & \text{if } x \geq 5. \end{cases}$

- a.  $f^{-1}(x) = \begin{cases} 4x & \text{if } x < 5, \\ 5/x & \text{if } x \geq 5. \end{cases}$       b.  $f^{-1}(x) = \begin{cases} -4x & \text{if } x < 5, \\ x/5 & \text{if } x \geq 5. \end{cases}$       c.  $f^{-1}(x) = \begin{cases} x/4 & \text{if } x < 20, \\ 5/x & \text{if } x \geq 20. \end{cases}$
- d.  $f^{-1}(x) = \begin{cases} x/4 & \text{if } x < 1, \\ 5/x & \text{if } x \geq 1. \end{cases}$       e. There is no inverse function

32. A group of people has a ratio of women to men of 11:10. The women have an average (arithmetic mean) age of 34 and the men 32. What is the average age of the entire group?

- a.  $32\frac{9}{10}$       b.  $32\frac{20}{21}$       c. 33      d.  $33\frac{1}{21}$       e.  $33\frac{1}{10}$