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Your impassioned commitment to teaching quality mathematics to each student is commendable. It is our sincere desire at the Indiana Council to Teachers of Mathematics (ICTM) to support you in this endeavor from the earliest levels of K-12 classroom experiences to the preparation of teachers in the higher education communities.

In representation of you, the ICTM provides multiple programs for the recognition and quality of best practices in mathematics teaching and learning. These programs include recognition awards for K-12 students, scholarships for teachers preparing for teaching mathematics, grants for special endeavors in mathematics classrooms, professional development of mathematics teachers, conferences with featured speakers from across the states, and mathematics contests for those who are reaching high levels of achievement in the mathematics field. It is our desire to equip you with the latest and best knowledge available in the field of mathematics. Join us by becoming a member to share and receive the benefits of our organization at **www.indianamath.org.**

Sheridan Rayl, ICTM President

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A Blizzard of a Value

Author: **Jonathan D. Bostic,** *Bowling Green State University*

This article originally appeared in Mathematics Teaching in the Middle School in February 2015.

"Who has been to Dairy Queen® and purchased a Blizzard?®" Ms. Bosetti asked her students. During the summer, Bosetti had seen many of her former and future students at the local Dairy Queen enjoying Blizzard desserts and wondered, "Which Blizzard size is the best value?" She used this context for a ratios and proportions task to give students experience modeling with mathematics.

Modeling with mathematics requires engagement in problem solving that is focused on realistic situations (CDE 2013; Kanold and Larson 2012). The "model with mathematics" text within the Common Core's Standards for Mathematical Practice (SMP) states: "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (CCSSI 2010, p. 7). Tasks and instruction that encourage students to analyze mathematical relationships, draw conclusions, and reflect and improve on end products (i.e., models) address SMP 4 (Fennell, Kobett, and Wray 2013). This characterization of modeling with mathematics brings up two key instructional questions: What should students engaged in modeling with mathematics be doing? What should teachers do to promote modeling with mathematics?

▸ COMMON CORE PROFESSIONAL DEVELOPMENT

Bosetti, a seventh-grade teacher in a rural Ohio school district, participated with 15 other middle school math teachers in a yearlong professional development (PD) program. While completing 100 face-to-face hours, the teachers focused on three goals:

1. To better understand the Common Core State Standards for Mathematics (CCSSM)

2. To better identify, select, and/or create tasks addressing the Content and Practice Standards

3. To make instructional changes that fostered mathematical proficiency around CCSSM

Figure 1

This example of a worthwhile task supported the learning of the content, promoted engagement, and fostered communication.

The Dairy Queen Dilemma
I love ice cream, and I love bargains. One day, I went to Dairy Queen to have a Blizzard. I saw four sizes listed with the following prices: The Mini Blizzard was $2.55, the Small Blizzard was $3.25, the Medium Blizzard was $3.80, and the Large Blizzard was $4.65. If I wanted to get the most Blizzard for my money, then which size should I buy? Prove that your selection is the best buy by justifying your answer.

The PD leaders recognized that Bosetti and her peers needed to experience problems that promoted mathematical modeling before they could design such a task for their own students. So that these teachers might have a way to reflect on what it means to model with mathematics, the teachers completed a task called the Airplane problem (modified from Case Studies for Kids 2013). In this task, teachers were given data from a paper airplane contest that showed the results of various attempts by several pilots to fly four different paper airplanes. Measurements with the plane's flight distance, distance from target, and time in flight were given for each attempt. The problem's objective was to use the data to give the judges of the contest the best way to determine which paper airplane was the "best floater" and which was the "most accurate."

Modeling with mathematics brings up two key instructional questions: What should students engaged in modeling with mathematics be doing? and What should teachers do to promote modeling with mathematics?

Bosetti and her peers worked on the problem for about two hours, frequently drawing on prior mathematics knowledge from various grade levels. Simultaneously, they figured out which quantities in the problem (e.g., time and distance) were important and any relationships between them. Each group wrote a report, delivered a presentation to their peers about their approach, and justified why their model for determining the winner was the best. Finally, each group reflected on possible model refinements. These problem-solving assumptions and actions are part of the mathematical modeling process (Fennell, Kobett, and Wray 2013). After completing the activity, Bosetti and the others realized that modeling with mathematics meant engaging students with real problems or realistic scenarios, talking with and about mathematical representations, and re-examining their mathematical models after receiving feedback from peers. Following the PD that included similar modeling tasks, Bosetti and others designed and implemented tasks that fostered reasoning and sense making related to CCSSM.

▸ CREATING THE DAIRY QUEEN DILEMMA

Bosetti followed the PD's approach (see Bostic 2012/2013) to construct the Dairy Queen Dilemma, a worthwhile task meant to engage her students in modeling with mathematics. (Names of all students in this article are pseudonyms.) Such a worthwhile task supports learning mathematics content, promotes engagement in mathematical practices, and fosters mathematical communication among students (NCTM 2007). This approach involved a four-step process: examining and reflecting on the mathematics standards (Content and Practices) of interest; considering possible mathematics curricular tasks that could be improved to become more like problems rather than exercises; reflecting on students' cultural contexts or motivating realistic events to frame the task; and then revising the task to produce a clearly worded worthwhile task. This task also addresses multiple Common Core Content Standards within the Ratio and Proportional Relationships domain, including "Recognize and represent proportional relationships between quantities" (CCSSI 2010, p. 48).

▸ CLASSROOM CULTURE

A key component for success with modeling-focused tasks is enacting appropriate norms for doing mathematics, talking about mathematics, and working collaboratively on mathematics projects. Early in the school year, Bosetti and her students established such productive classroom norms as these: "We will always look for another way to solve a problem." "We will always justify our work," and "We will critique the idea rather than the person." Having established these norms, Bosetti felt confident that her students were prepared for the Dairy Queen Dilemma.

▸ THE DAIRY QUEEN DILEMMA

Bosetti told her students she was recently at Dairy Queen and wondered which Blizzard size was the best value, assuming that calorie content was unimportant. Several students suggested that the large Blizzard was likely the best value, possibly drawing on prior knowledge that larger sizes tended to be better values. Bosetti then distributed copies of the problem (see fig. 1), and students self-selected groups of three or four. She then gave each group a set of four cups, one cup of each size, as well as one measuring tape. Most groups started problem solving by measuring the cups' diameters and heights. Discussions focused on how to measure the cups (i.e., inside or outside). Matthew, like most students, measured the diameter of the base and noticed that the bases of the mini cup and large cup were smaller than those of the small and medium cups. "Something isn't right here!" he said, making a puzzled expression. Many students, like Matthew, remeasured their cups to confirm this unusual finding. Matthew then shared his finding with Bosetti. When she asked why, Matthew replied, "I don't know," then continued problem solving with his group.

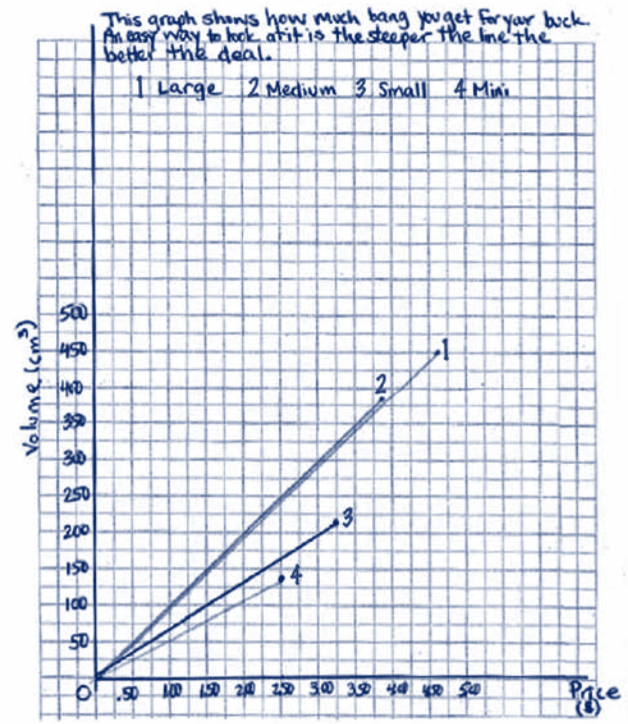
Multiple groups made a second observation: The bottom and top bases of the cups were not the same measurement. Bosetti stopped the class and asked, "What should we do?" Emma suggested that they assume that each cup's base was identical because the difference was less than half a centimeter. Bosetti asked the class, "Do you agree? Do we want to assume that?" Students nodded in agreement and went back to work.

A short time later, one group asked Bosetti, "How full is the Blizzard cup?" Again, she stopped the class and asked students how to proceed. Matthew and others thought that they might assume that the ice cream was level with the top of the cup. This assumption made modeling easier, but Megan reminded everyone that the local Dairy Queen usually made Blizzards that spilled over the top; hence, their mathematical models might not be completely accurate. This assumption offered some degree of precision and accuracy that seemed reasonable for the problem.

After circulating around the room, Bosetti noticed that students were ready to share their problem solving with peers. She purposefully selected students from three groups to present strategies that involved graphs, tables, and symbol manipulation. They shared their ideas by (1) displaying their written work for the class, (2) explaining what they did and why, and (3) justifying their approach and solution. Three groups' work and ensuing class discussions are shared.

Figure 2

Toni's graph explored "how much bang you get for your [Blizzard] buck."



▸ A GRAPHICAL APPROACH

One group of students used a graph to model the mathematical elements embedded in the situation. Toni, placing her graph (see fig. 2) under the document camera, said that her group tried to create one coordinate point for each cup size so that they might create a straight line with the origin.

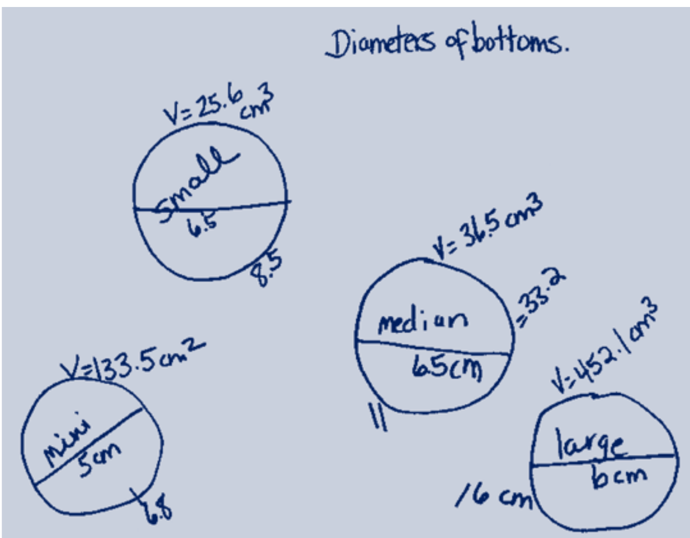
Her group put price in dollars on the x-axis and volume in cm^3 on the y-axis. This approach created four line segments, one for each cup size. Toni added that the steepest line indicated the best value, yet she had difficulty explaining why this approach worked, saying, "It just seemed like the right way." Toni's peers commented that her justification needed some work but that the model seemed reasonable. Toni closed by adding that the medium was the steepest slope; however, she agreed that more information might clarify her group's position. For instance, it might help if she labeled the endpoints of each segment. Bosetti asked if her group wanted more time to think and reflect on their model; the group agreed to explain their thinking later.

▸ A PICTORIAL APPROACH

Sean placed his group's work, which involved a pictorial approach combined with some calculations, under the document camera (see fig. 3). "Each circle represents a different cup size. We measured the diameter, then calculated the circumference, and finally found the volume for each cup." He then shared that his group compared price and volume of the cups during their small-group problem solving before making the whole-class presentation. This comparison was not evident in their written work. The ratio for the large and medium were close. They determined that the large was 97.2 cm^3 per dollar, whereas the medium was 96.1 cm^3 per dollar. On the basis of these findings, they felt that the large size was the best value because "you get more Blizzard per dollar in the large." As soon as Sean ended his presentation, several hands were raised. Emily asked Sean, "I got different ratios. How did you measure your cups?" Sean showed the class by holding the ruler to the base of the cup to measure the diameter and then the edge of the cup to determine the height. Emily replied, "That's how I did it, but I got different volume measurements." Bosetti asked Emily if she would like to share her work next so they might compare it.

Figure 3

Sean's diagrammatic approach to the Dairy Queen Dilemma compared price and volume of the cups, although it was not evident in the drawing.

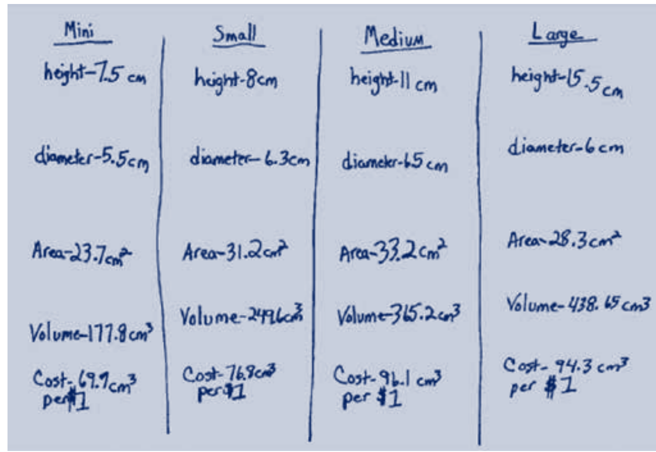


▸ A TABULAR APPROACH

Emily's group created a table (see fig. 4) to organize information about the cups. Emily and her peers discussed the information they gathered (i.e., cup height, diameter, area of cup base, volume of cup, and cup price) and why that information was necessary to answer the problem. Her group concluded that the largest unit rate, the medium Blizzard (96.1 cm^3 per dollar), was the best value. Emily added that the large Blizzard (94.3 cm^3 per dollar) was just slightly less than the medium. Sean commented that his height measurement for the large size was different, which resulted in different volume and ratio values.

Figure 4

Emily's group concluded that the largest unit rate, the medium Blizzard (96.1 cm^3 per dollar), was the best value.



Emily and Sean exchanged ideas and tried to show each other in front of the entire class that one individual's measurements were correct while the other was incorrect. Breanna interrupted the exchange and addressed Bosetti, "They might have measured differently, so their numbers might be slightly different." Bosetti pressed Breanna about what she meant by "measured differently." Breanna described that it was difficult to be precise with the measuring tools they had, which might lead to rounding errors. Bosetti asked, "Is it possible that if we had better tools then we might be able to better answer this question? You know, is it 15.5 or 16 cm?" Everyone nodded in agreement. "Given what we have, is it possible to have different answers about the best value?" The class answered yes, that the exact height of the large cup was between 15.5 cm and 16 cm. "Rounding your answer down to 15.5 or up to 16 cm impacts your final answer, doesn't it?" asked Bosetti.

Students nodded in agreement because both groups' answers were correct, based on their problem-solving approach. Given their tools, it was reasonable to conclude that either the medium or large was the best value; however, the work had to justify the conclusion. Sean and Emily concluded that better measuring tools might help determine the best value by alleviating the rounding error.

Productive classroom norms include looking for another way to solve the problem; justifying the work; and critiquing the idea, not the person.

Emily then explained, "The medium is the best value. But if you wanted more Blizzard, then the large wasn't a bad deal." This comment raised an interesting point that Bosetti expanded on: "If I want a large Blizzard, is it much worse in value than the medium?" "Not really," Matthew said, "if you want the large Blizzard, then get it. It's close in volume per dollar to the medium Blizzard's ratio." Clearly, students were wrestling with the real-life implication of this task: If an individual wants more, then he or she might be willing to pay more.

The next day, Bosetti opened her class by asking Toni's group to share their ideas. Toni placed her work (see fig. 2) under the document camera and said, "We measured the cups like Sean's group. Then, we found the ratio of a cup's volume to dollars and simplified it to find the unit rate. Then, we compared those numbers. The biggest number [the unit rate expressed as volume per dollar] was the best value. We think the medium is the best value, just slightly better than the large. You can see this in our graph because the red line medium) is on top of the blue line (large), which means the medium is the best value. We understand how Emily's group thought the large was a better value."

▸ LESSONS LEARNED FROM THE DAIRY QUEEN DILEMMA

Following the Dairy Queen Dilemma, Bosetti abstracted some key ideas for students from the experience. First, she reminded students that numerous effective ways are possible to determine whether one ratio is greater than another. Although symbolic approaches may be efficient, Toni believed that the graphical approach easily showed her which ratio was greater. Bosetti hinted that Toni's idea of comparing unit rates (slope) would come up again later in mathematics.

Second, students added that they learned how to compare ratios while solving a realistic problem. They felt motivated to determine the best value and did so in a logical manner: by comparing unit rates. Bosetti did not lecture during her lesson at any time. The problem itself provided an engaging context to foster perseverance while problem solving. Students were able to recognize and later represent the proportional relationships to determine the best Blizzard value.

An important feature of Bosetti's instruction was that it engaged students in modeling with mathematics using multiple representations (i.e., tables, graphs, and diagrams). Students were able to frame their thinking using appropriate representations and effectively compare the unit rates for each cup size. The lesson also gave students like Toni a chance to re-evaluate their thinking and have a second chance to describe their problem solving.

Finally, students learned a valuable lesson about using appropriate tools and about limitations of precision. Many students wished for more accurate tools that measured tenths of a centimeter rather than half centimeters. They wanted to have the answer rather than an answer, which was more typical of other problems they solved. Bosetti used this opportunity to share important conclusions from the activity: Sometimes there are two answers to the same problem, and both might be correct. Or, if two or more individuals' problem-solving approaches are justified adequately and limitations of those approaches are recognized, then multiple solutions are possible.

▸ HOW A BLIZZARD CAN HELP MODEL MATH

The Dairy Queen Dilemma engaged students in making sense of problems and persevering in solving them, modeling with mathematics, using appropriate tools strategically, and attending to precision (CCSSI 2010).

Students felt that this modeling task was a fun way to use mathematics to make sense of real-life phenomena.

One student wondered aloud whether the medium size might be the best value compared with other cup sizes, like those found at fast food restaurants or convenience stores. "I never thought that the medium was the best value [at Dairy Queen]. I assume it's always the biggest one," said Breanna. Bosetti asked Breanna and others to explore this question on their own. Bosetti said that the encouragement to create and implement the Dairy Queen task in her classroom as well as the feedback from her PD instructors and peers gave her confidence to write and enact other tasks that drew on students' interests and the local community:

"Teaching was already fun for me, but now it's even more fun. Students are excited and engaged in learning math. I don't worry about [explicitly] teaching math when I do modeling problems like the Dairy Queen Dilemma because students are doing mathematics."

CCSSM Practices in Action

SMP 1: Make sense of problems and persevere in solving them.

SMP 4: Model with mathematics.

SMP 5: Use appropriate tools strategically.

SMP 6: Attend to precision.

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Three Strategies for Opening Curriculum Spaces

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Imagine the following scenario: A third-grade teacher opens the teaching guide for her district-mandated textbook to a multi-digit subtraction lesson. The lesson shows the standard algorithm for subtraction, using the problem 377 – 187 = ? This example is followed by ten practice problems and, at the end of the lesson, two word problems. Reflecting on her students' participation in a mental math routine earlier in the week, the teacher knows that her students can think about these problems in at least three other ways (see table 1). She also knows that, by the end of the lesson, most of her students will assume (correctly) that the word problems involve subtraction situations and will automatically apply the standard algorithm to solve those problems.

How can this teacher present a more meaningful, engaging lesson that builds on what she knows about her students' understanding of subtraction and their prior experiences with subtraction, both in and out of school, while still adhering to her district's mandate to use the textbook?

▶ COMPETING EXPECTATIONS

Many teachers confront this tension between using published curriculum materials and teaching in ways that are responsive to children. Teachers are often expected to use a particular mathematics curriculum series, but they still want to be able to build on and connect to children's multiple mathematical knowledge bases (MMKB). Children's MMKB include children's mathematical thinking and children's home- and community-based mathematical funds of knowledge (Carpenter et al. 1999; Gonzalez, Moll, and Amanti 2005; Turner et al. 2012). Children's experiences using mathematics as part of home or community activities, as well as family practices that involve mathematics, are all part of children's home and community-based mathematical funds of knowledge. Children's mathematical thinking includes the multiple strategies that students use to solve problems, if given the opportunity, as well as common confusions or misconceptions that children might have. In several of its standards, the Common Core State Standards for School Mathematics (CCSSM) calls for students to use multiple solution strategies (CCSSI 2010). Students make sense of problems and develop multiple solution strategies by connecting problems to their own experiences both in and out of school and by using and building on all of their MMKB.

TABLE 1

Reflecting on her third graders' participation in a previous mental math exercise, the teacher knows that her students can think about problems in at least three ways other than what their math book presents.

Solutions for 377 – 187 = ?

Solution A	Solution B	Solution C
300 – 100 = 200	187 + 3 = 190	"I know 377 take away 200 is 177. 177 is ten less than 187, so 377–187 must be 190."
70 – 80 = –10	190 + 10 = 200	
7 – 7 = 0	200 + 177 = 377	
200 – 10 + 0 = 190	177 + 10 + 3 = 190	

▶ SMALL ADJUSTMENTS, BIG DIVIDENDS

Eliciting and building on children's MMKB while using mandated curriculum materials is a significant challenge, because curriculum materials often focus on single strategies for solving problems and single meanings for problem contexts. In this article, we present three strategies for making small changes to curriculum materials that can open spaces for eliciting, building on, and connecting to children's MMKB. Each strategy requires small adjustments that are both mathematically impactful and reasonable for teachers to implement.

We illustrate several of these strategies using a lesson from Grade 4 Everyday Mathematics (UCSMP 2007, pp. 406–11). We chose this lesson because Everyday Mathematics promotes Standards-based content and practices and is used widely in schools and classrooms across the country. Our point is that all curriculum materials, even high-quality materials, require teachers to make changes to open spaces for children's MMKB. In part, this is because any set of published curriculum materials must be written for a generic classroom, so teachers have an important role in adapting and using the materials to meet the strengths, needs, and experiences of specific children.

The stated objective of the Everyday Mathematics lesson is to "guide the exploration of strategies to solve equal-grouping division number stories" (UCSMP 2007, p. 406). At the beginning of the lesson, in the Math Message, students read the following problem:

A box holds 6 chocolate candies.
How many boxes are needed to hold 134 chocolates?

The teacher is then directed to ask several students to share their solutions. Four possible strategies are provided in the teaching materials to give teachers an idea of what to expect from their students:

1. Direct modeling

2. Drawing a picture

3. Break[ing] 134 into smaller "friendly numbers"

4. Thinking of the problem as a multiplication problem with a missing factor

Teachers are expected to build on and connect children's multiple mathematical knowledge bases (MMKB).

The next three pages in the teaching guide direct teachers to explain the "multiples strategy" as "one way" to solve these types of division problems. The multiples strategy involves using decade multiples to find the answer to a division problem. For example, to solve the problem in the Math Message using the multiples strategy, students would figure out how much ten sixes, twenty sixes, and thirty sixes equal and then use that information to find the solution to the problem.

After the explanation of the multiples strategy, students receive several problems for their math journals. The teaching guide states that teachers should "encourage students to use a variety of strategies to solve the problems …" (p. 410), but the format of the journal pages supports the use of the multiples strategy. The pages have little room for students to explore other solution strategies, including those that were shared after the Math Message. Below, we suggest three strategies for opening spaces for children's MMKB in curriculum materials, and we provide examples of those strategies, using the Everyday Mathematics lesson. We focus on lesson changes that stay consistent with the stated lesson objective while opening spaces for children's MMKB.

Strategy 1. Rearrange lesson components

Most curriculum lessons have several components, including opening routines or messages, a variety of student tasks, differentiation suggestions, and homework. In reviewing many textbooks, we found that more open spaces tend to be located on the periphery of lessons, including opening messages, textbook margins, teacher notes, and differentiation ideas. Often, teachers can open spaces for children's MMKB by moving these components around or omitting some of them and focusing on others. In general, the goal is to find those components that focus on (1) having students make connections between the task and their prior knowledge and experiences, (2) providing support for students to develop their own strategies, and (3) encouraging students to share and explain their strategies. After these components have been identified, strategies for rearranging them to open curriculum spaces and support children in making meaning of the mathematics could include the following two options.

▶ Front-load problem solving

In many textbooks, the tasks that demand complex problem solving are located in the textbook margins or at the end of the lesson as an application or as a problem-solving section. Or they might appear in the beginning of the lesson as a warm-up activity, as in the Math Message component of the Everyday Mathematics lesson described above. Or they might be located in teaching notes related to homework, enrichment activities, or advanced learner sections that are not part of the main lesson. Focusing the majority of the lesson on these problem-solving tasks and making these the "main" lesson will open spaces for children's MMKB, while still maintaining the mathematical goal of the lesson. The key to front-loading problem-solving tasks is to engage children in those tasks before introducing a preferred solution strategy.

▶ Cut components

Consider omitting sections of a lesson that tell, direct, or show children how to make sense of and solve problems. Use this extra time to make connections to children's MMKB, exploring ways in which children's previous experiences and home- and community-based mathematical funds of knowledge can be leveraged to support new solution strategies. For example, consider using the extra time to listen to what children already know about the lesson topic and when and how they have encountered that topic outside school.

In the Everyday Mathematics lesson, we would rearrange the curriculum by omitting the teacher's explanation of the multiples strategy but retaining the Math Message, strategy sharing, and some kind of independent practice (although with adaptations, described below as part of strategy 2). We would begin the Math Message by asking students about their out-of-school experiences with items that come in packages, such as boxes of chocolate. Students could discuss how many chocolates fit in boxes of different sizes and shapes, the arrangement of chocolates in rows and layers, other items that come in boxes, and so on. We might ask students how they would know or figure out how many chocolates were in ten boxes or twenty boxes and how they might decide to share those chocolates with one or more people. Students could discuss their experiences of sharing items fairly, including strategies they have used to make sure each person receives the same amount. We might also ask students how they could organize chocolates into boxes (i.e., different dimensions of rows and columns of twelve or twenty-four chocolates) and how they could predict, estimate, or figure out how many boxes they would need for a given number of chocolates. By re-arranging the curriculum in this way, the space opened in the Math Message is maintained throughout the entire lesson and children are able to develop and explore a variety of strategies, as stated in the lesson objective, using their prior experiences with the content.

Strategy 2. Adapt tasks to open spaces for children's MMKB

Adapting textbook tasks in ways that open spaces for children's MMKB—particularly children's mathematical thinking—is often possible with specific strategies, such as those that follow.

▶ Adjust numbers or offer choices

Adjust the numbers in the problem or provide multiple number choices. By adjusting the number choice, you can open access to struggling learners and fast finishers while maintaining the cognitive demand and mathematical goals of the lesson. All students are able to work on the same mathematics but in ways that connect to their own prior knowledge. This often can be done by providing multiple number choices for a single problem and allowing students to work on the numbers that are "just right" for them.

▶ Encourage multiple representations and strategies

Encourage multiple representations and solution strategies; having more than one tool in their toolbox is important for students. By asking them to solve a problem in two different ways or to use multiple representations, you can increase their capacity to solve problems, their practice justifying their solutions, and their ability to compare and contrast solutions to deepen mathematical understanding—all important mathematical practices highlighted in CCSSM.

In the Everyday Mathematics lesson, the first journal page presents three story problems with significant scaffolding for using the multiples strategy to solve them. The first problem reads as follows:

José's class baked 64 cookies for the school bake sale. Students put 4 cookies in each bag. How many bags of 4 cookies did they bake? (p. 409)

Instead of having students work on all three problems, we might provide students with only this one problem along with number choices tailored to student needs. We could present the problem without the scaffolded activity sheet and encourage students to use at least two strategies or representations to solve the problem. This problem could be solved either after or instead of the Math Message:

José's class baked _____ cookies for the school bake sale.
Students put _____ cookies in each bag.
How many bags did they make?

A. (24, 4)

B. (64, 4)

C. (180, 6)

D. (276, 6)

E. (191, 5)

In this example, we used only one of the problems on the workbook page, but we included the number choices provided in all three problems. We also offered two additional number choices. Generating appropriate number choices for a classroom can prove difficult when you do not know a particular group of students. However, for the purposes of this example, we assumed that some students might find 64 ÷ 4 difficult. Therefore, we included an easier number choice. We included the third number choice to provide a transition to the fourth number choice so that the leap between the choices is not as big. If number choices of higher difficulty are needed for a particular class, you could supply them. In summary, we are saying that number choices can be used to differentiate a curricular task and fit the needs of any classroom. (See Land et al. [forthcoming] for more examples of number choice as a differentiation strategy.)

Strategy 3. Make authentic connections

Look critically at the real-world contexts presented in the textbook. Do these contexts actually help your students make sense of the mathematics? Are they meaningful to the students in your classroom, and do the mathematical practices in the problem or task actually connect to an activity in which children might engage in that real-world context? The word problem given in the student workbook of the Everyday Mathematics lesson and adapted above reflects experiences students might have in or out of school, but these may not be experiences that your students have had. Are there other experiences or situations that would be more authentic? One way to answer this question is to ask students to find multiplication and division situations at home or in the community, write about them or take a picture, and share them with the class. Possible situations include the number of children playing in a soccer club on a Saturday morning or the numbers associated with planning a large event, such as a wedding, quinceañera, or family reunion. These situations can be used as contexts for future word problems. A related adaptation, as suggested above, would be to begin the lesson by having students share examples of items that come in packages in which each package has the same number of items. Then, consider situations when it might be useful to know how many packages are needed or how many packages can be filled with a given number of items.

▸ Curriculum spaces

In our work, we have found that curriculum materials have "spaces" for students to make sense of mathematics, but those spaces are often closed when the materials require (or strongly suggest) that students use a particular strategy or when the materials present solution strategies before connecting to children's MMKB. We have developed a set of strategies for teachers to use to open curriculum spaces through relatively small changes to elementary mathematics curriculum materials. We think of these curriculum spaces, or third spaces (Moje et al. 2004), as places for teachers and students to create bridges for using children's MMKB to support school mathematics learning.

Enacting these strategies is likely to be easier or more difficult depending on the particular materials. However, these strategies can be applied to any lesson by focusing on tasks that provide openings for children's MMKB and by engaging students in rich discussions. Many curriculum series include opportunities for "higher-order thinking" or "problem solving" at the end of lessons. Choosing one or two of these problems and opening the lesson by allowing children to develop and share strategies for solving those problems provides an opportunity for teachers and children to build on children's MMKB in ways that closely align with CCSSM. Similarly, almost all lessons can be opened by eliciting children's out-of-school stories and experiences with the mathematical content and task, again opening space for children to make sense of the mathematics through their own MMKB (See also Butterworth and Lo Cicero 2001). Curriculum spaces—or potential curriculum spaces—exist in all curriculum materials; and with small changes, can be powerfully leveraged to support the learning of all children.

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INSIDE THIS ISSUE

2015 Elections

The Indiana Council of Teachers of Mathematics (ICTM) Board of Directors 2015 fall election includes the following positions:

- **President-Elect**
- **Elementary South**
- **Middle School North**
- **High School Central**
- **College South**

Each position is a three-year term, except the President-Elect which is a four-year commitment. The Board meets about four times a year (quarterly) in Indianapolis. Board members also serve on subcommittees that involve a time commitment in addition to the regular board meetings.

If you are interested in serving on this Board, please complete and submit the self-nomination application located on indianamath.org by August 3, 2015. Directions for submitting your application are at the bottom of the application form. If you are not sure which region of the state you represent, please see the state map on the website: indianamath.org. If you reside in one county but are employed in another, you may select the county you wish to represent.

If you have any questions, feel free to email Jane Mahan, ICTM Nominations & Elections Chairperson, at jane.mahan@evsc.k12.in.us