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Developing Mathematics Specialists: Stories of Teacher Leadership

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Many schools and districts across the nation are using mathematics specialists, teacher leaders, or coaches to improve teaching and learning in mathematics (McGatha, Davis, & Stokes, 2015). Recommendations for the preparation of these specialized professionals include the development of strong mathematical knowledge for teaching, pedagogical knowledge for teaching mathematics, and knowledge and skills for leadership (Association of Mathematics Teacher Educators, 2013). This article describes the journeys of six elementary- and middle-school teachers who worked to develop such knowledge while enrolled in a graduate course in mathematics teacher leadership. Their stories may be of interest to teachers, school principals, and teacher educators.

Throughout their semester together, the teachers read and discussed the following two books published by the National Council of Teachers of Mathematics (NCTM): *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014) and *The Elementary Mathematics Specialist's Handbook* (Campbell, Ellington, Haver, & Inge, 2013). In order to put some of the ideas from their reading into practice, each teacher conducted a school review with their school principal using the *PRIME Leadership Framework* (National Council of Supervisors of Mathematics, 2008) and designed and implemented both a professional development workshop for their school colleagues as well as another teacher leadership activity of their choice.

As the teachers took on newer roles as mathematics teacher leaders, they remained mindful of the importance of relational skills. The *PRIME Leadership Framework* defines a *relational leader* as one who listens to colleagues without interrupting or judging, respects confidences without betraying secrets or private conversations, and practices empathy through deliberate inquiry (National Council of Supervisors of Mathematics, 2008). The teachers share their own stories regarding relational mathematics teacher leadership and describe the professional development workshops and other leadership activities they conducted in their schools. Their stories appear in alphabetical order, according to the teachers' last names.

The Teachers' Stories

Melissa

Melissa possesses a passion for teaching both junior high students and mathematics. She teaches Algebra to eighth graders at an independent Catholic school in Memphis, Tennessee. Melissa's school had no written documents outlining the standards and objectives for each mathematics class. Working without an overall curriculum guide felt like walking a tight rope without a net; so, Melissa and a seventh grade teacher and colleague felt compelled to develop the documents. The curriculum plans they created would serve as the foundation for the teaching of mathematics at the junior high level. Initially, the plans included standards, learning targets, and a loose pacing guide. To accomplish the task, Melissa requested professional development release time for herself and her colleague. The two colleagues spent a day discussing the data from the eighth graders' high school placement test as well as that from the seventh graders' achievement test. Using this testing data and Tennessee's state standards, the teachers developed curriculum plans for each of the math classes taught in the seventh and eighth grades.

In addition to developing the school's math curriculum, Melissa led an hour-long workshop for her colleagues that focused on formative assessment, per the request of her school's administrators. Two of the three goals she set for her workshop were met; the one not met involved her colleagues' willingness to brainstorm and share ways to use each formative assessment technique example. Melissa anticipated that her fellow teachers would be enthusiastic and open to discourse. Her colleagues were open to listening to her share but were not willing to actually brainstorm in the way Melissa wished. The unmet expectations initially led Melissa to believe her workshop had been unsuccessful. The feedback on the workshop evaluations, however, was positive; so, after a few days of reflection, Melissa realized her colleagues simply did not feel comfortable sharing. *The Elementary Mathematics Specialist's Handbook* (NCTM, 2013) states that teachers who do not feel secure in their mathematical ability are usually not willing to share in group

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About the Journal

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The *Indiana Mathematics Teacher* is a peer-reviewed publication of the Indiana Council of Teachers of Mathematics. The *Indiana Mathematics Teacher* provides a forum for mathematics teachers from pre-kindergarten through college to present their ideas, beliefs, and research about mathematics teaching and learning. We are currently seeking manuscript submissions, and welcome them from preK–12 teachers, university mathematics educators, professional development providers, graduate students, and others with a vested interest in mathematics education. Manuscripts should be written for an audience of K–16 mathematics teachers and should be limited to approximately 1500–3000 words. For more information and full submission guidelines see *http://ictm.onefireplace.org/* or contact the editors at *djmohr@usi.edu* and *rhudson@usi.edu*. If you are willing to serve as a peer reviewer to provide feedback on potential articles, contact one of the editors.

settings. Next time, Melissa plans to actually have the participants model an example of each formative assessment technique. Her intent is to use different math topics and vary the grade level of each assessment example.

Danielle

Danielle teaches at an independent girls' school in Pasadena, California. Over the past 23 years of her career, Danielle has taught a variety of grade levels, spanning kindergarten to sixth grade. She has instructed students in all elementary subjects but has solely taught mathematics for the past thirteen years. She is very passionate about girls' education and mathematics.

Danielle recently planned two leadership activities to present at her school. One activity was called "Mad for Math Day." She invited the parents of her fourth grade students to take part in an interactive morning that celebrated their daughters' achievements in mathematics. Danielle had her students highlight solid mathematical thinking and communication for their parents by having them participate in rich problem solving tasks and games that fostered such skills. The fourth graders modeled these activities and games as the parents completed the tasks. One activity was called "Alternate Algorithms" and involved the students teaching their parents a variety of algorithms such as partial products multiplication, lattice multiplication, and partial quotients long division. Parents were overheard saying, "I never knew multiplication could be done this way," or " I love seeing my daughter so comfortable with numbers!" Needless to say, "Mad for Math Day" was a huge success.

Danielle also organized a curriculum mapping professional development workshop for her school colleagues. She decided to teach her sixth though eighth grade math colleagues how to create a curriculum map of the skills and content taught at each grade level. The purpose behind this professional development session was for her colleagues to use the completed curriculum map in order to compare the content and skills taught at her school to those outlined in the Common Core state standards. Danielle chose curriculum mapping as her topic for the workshop because it aligned with one of the guiding principles for school mathematics—curriculum. Her hope was to improve upon an already-strong math program at her school. Creating an ever-changing curriculum map for the school would help foster an even stronger math program for the girls.

Kelli

Kelli teaches third grade in Fort Wayne, Indiana. From the beginning of her career, Kelli has been frustrated with the lack of support in teaching mathematics. Her traditional mathematics textbook lacked higher-level, engaging mathematics tasks. So eight years ago, Kelli began researching best practices as well as high-quality resources she could use in her classroom. She shared the information over the years with her colleagues and led book studies with fellow staff members. Opportunities to share her research led her to further her education in mathematics leadership.

Kelli recently stepped out of her comfort zone by leading a professional learning opportunity at her school. Her workshop was based on the book *Good Questions for Math Teaching: Why Ask Them and What to Ask* (Sullivan & Lilburn, 2002). This book provides two strategies for writing and asking good, quality questions. The workshop participants learned about the importance of asking good questions, completed an activity to determine the quality of the questions asked in the textbook adopted by the district, and then practiced the two strategies as described in the book for asking good questions. The workshop was well received by the participants, as several of them began immediately implementing the strategies.

Kelli was also challenged to increase her leadership within her grade-level team. Her team had never before worked together and struggled to plan high-quality math units and lessons. The team lacked goals and focus. Kelli began to implement the coaching techniques she learned in her mathematics teacher leadership class. The team began to work more closely but was still unfocused. Kelli approached the team about working through a curriculum map to prepare them for the next school year. The team members were excited to have a plan to create the curriculum map, knowing it would allow them to be more focused in their teaching the following year. The first step to creating the curriculum map was to establish a pacing guide. According to *Principles to Actions* (NCTM, 2014), it is important to remember that while pacing guides help teachers sequence the learning and make connections in their teaching, the guides can also cause teachers and students to "feel rushed" (p. 71). In order to not let such negatives outweigh the importance of the curriculum map, the team members did not create a calendar; instead, they created a table dividing each domain into an approximate number of weeks on which to spend teaching the material. The team felt more confident in their plan after creating the pacing guide and committed to continue working on the curriculum map for the remainder of the school year. Moving forward, Kelli plans to continue to cultivate her leadership skills by seeking out professional development opportunities. Her ultimate professional goal is to either become a math coach in an elementary school or hold a district leadership position.

Maggie

Maggie is a math interventionist for kindergarten through sixth grade students in Hagerstown, Indiana. She sees students from all grade levels daily and enjoys helping them strengthen their math skills while also developing more positive dispositions about math. She has found that there are a lot of leadership opportunities for those willing to lead. Identifying areas of need and setting specific, achievable goals has proven to be a productive way for her to lead change and build foundations for success.

Working especially in intervention, Maggie has seen a multitude of positive outcomes from using different strategies and discourse in mathematics classrooms. Engaging students in their learning and building conceptual understanding by way of non-traditional algorithms is a point of knowledge and passion for Maggie, and she chose this as the topic for a professional development workshop she facilitated for her colleagues. Her workshop was intended to model a classroom environment; teachers solved various mathematical problems and then engaged in conversations about their strategies and thinking. They were challenged to solve the problems in different ways and weigh the pros and cons of the different strategies, all while explaining what they understood about the mathematics in each strategy. They tried each other's strategies and deepened their own content knowledge and, as a result, embraced new ways of thinking about different math concepts. It was a very positive experience; teachers found that the engagement and interest they had while solving problems, trying new strategies, and discussing methods with their peers could also translate to the positive environment they, as educators, create for their students.

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Maggie's school also aimed to increase family involvement and promote mathematics as an interesting and fun subject in which to engage. To support such goals, Maggie led the school's math committee in hosting a family math night. Held in conjunction with the spring book fair, there was a constant stream of families rotating around different stations. Geometry circle folding, measurement and fractions trail mix, math magic demonstrations, card and dice fact fluency games, estimation stations, technology games and presentations, and a beat-the-teacher problem solving station were all engaging opportunities for students and their families to enjoy mathematics. It was a fun night enjoyed by all--but perhaps most enjoyed by the fifth and sixth grade students who helped lead the activities for students and their families.

Maggie notes that the educational system is currently undergoing many changes in standards, curriculum, assessments, rigor, expectations, and teacher evaluations. She acknowledges that these changes may incite feelings of anxiety, but she also sees them as opportunities to lead positive change.

Mariah

Mariah teaches sixth grade math in Plainfield, Indiana. As a new graduate student, the mathematics teacher leadership class challenged her to take on new responsibilities. She worked with her school colleagues to create a common assessment for incoming sixth grade students. This assessment was intended to help determine which standards the teachers needed to reteach or include in a spiral review. Mariah also developed a presentation to share examples of ninety-minute math block structures available to her and her colleagues. She presented two different formats, and she and her colleagues determined the best way to set up their classrooms next year with a new time allotment.

Sarah

Sarah teaches kindergarten in Midvale, Ohio. She has taught only three years, but she realizes the potential leadership that every teacher has to offer—regardless of age or experience. Prior to this class, Sarah had never been interested in leadership. The thought of getting up in front of others and talking about educational policies, best practices, or data was not what she envisioned for herself. However, working with people was something that interested her, and she began to consider the possibility of relational leadership.

Sarah began talking about issues in mathematics with her colleagues. She portrayed herself as a listening ear when problems arose. She took the time to listen to complaints, even if they seemed unfounded at times. This began to show her co-workers that she was trustworthy. After she had an audience, Sarah planned a professional development workshop in the area she knew best: lower elementary. She held two short workshops before school—one for kindergarten and one for first grade—to address an area of need that the principal and teachers had determined: assessment. She began each workshop by helping teachers examine a strand of standards for grades K-2, and she then presented an assessment task that could be used as a common assessment. Sarah's intent was to make the experience meaningful. By giving teachers things that they could take away and implement, Sarah attempted to make herself useful.

Sarah also worked with another teacher to design a curriculum map that included assessments, curriculum units, and additional topics to be covered throughout the year. The motive behind this was to increase collaboration in her grade level, which was sorely lacking. There was very little collaboration beyond the once-a-week meeting, during which no planning took place. With a new team member coming on board next year, Sarah and her principal both agreed that promoting collaboration would be a positive change that could easily take place at this time.

By taking small steps of initiative, in her relational way, Sarah began to see herself as a leader. She knows there is a need for mathematics leadership at her school, but she is not sure if she will become a mathematics specialist. For now, she is content with engaging in small leadership activities and continuing to develop relationships with her coworkers.

Conclusions

These stories of mathematics teacher leadership are inspiring. They present a vision of elementary- and middle-school teachers who have delved into new ideas about mathematics teaching and learning and have courageously shared those ideas with their school colleagues. They help us see some of the potential for utilizing specialized knowledge in the forms of working with colleagues to develop curriculum and assessments, exploring problem solving strategies and questioning techniques, and working with students and their families to promote interest in mathematics. Other teachers may be interested in doing this kind of work. School principals may recognize the benefits of utilizing the expertise of mathematics specialists, teacher leaders, or coaches. Teacher educators may see opportunities for developing teachers' skills in mathematics teacher leadership.

Any number of these six teachers may remain in their classrooms, using their specialized expertise to foster children's mathematical thinking. Others may decide to focus on helping other teachers develop their knowledge and skills for teaching mathematics. As each teacher finds her own set of strengths, we celebrate such efforts to improve mathematics teaching and learning in schools at all levels.

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Seeing Spots and Developing Multiplicative Sense Making

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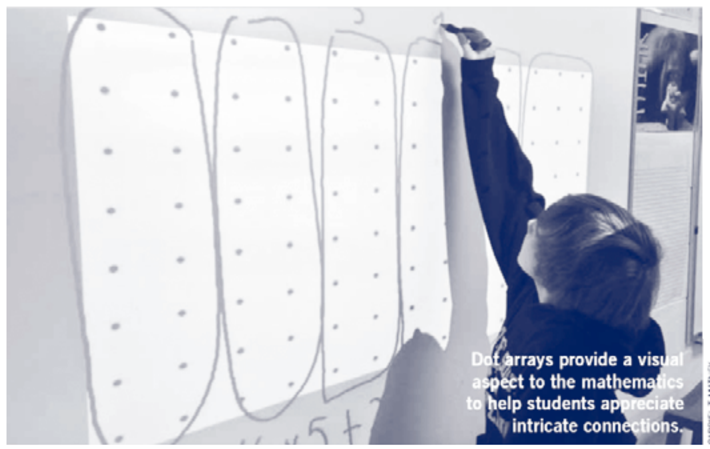
This article originally appeared in Mathematics Teaching in the Middle School in October 2013.

In Damien Hirst's Capric Acid Amide (2003), the viewer sees a lot of dots. This piece of art contains 11 rows and 14 columns of colored dots. One may ask, "What would be the most efficient way to count them all?" Is it best to think of these dots as 11 groups of 14 and add 14 eleven times?

Or are there other, more efficient ways to count the dots? When students are asked to represent efficient multiplication methods and present multiple strategies, including partial products and the standard algorithm, dot arrays similar to Capric Acid Amide can be used to engage students visually.

Dot arrays provide opportunities for students to notice structures like commutativity and distributivity, giving these properties an image that can be manipulated and explored. These images also connect to ways that we organize discrete objects in everyday life.



Dot arrays provide a visual aspect to the mathematics to help students appreciate intricate connections.

It is likely that you have encountered many rows and columns at your local grocery store (see figure 1). Because store managers like to keep their shelves looking fully stocked, items are often layered five deep. If the cans on the shelves in the photograph are currently only three layers deep, how many cans will be needed to ensure that the shelves are fully stocked? What properties and strategies involving multiplication could be used to efficiently find an answer? We will describe how we engage students in making sense of these multiplicative situations by using dot arrays.

Fig. 1

A display of cans on a grocery store shelf is a real-life model of an array representation.

As middle school teachers, we have noticed that many students begin sixth grade with a limited understanding of multiplication and its properties. That is not to say that they do not know how to correctly perform two-digit and three-digit multiplication using the standard algorithm. Rather, they lack a sense of flexibility, efficiency, and accuracy (Russell 2000). When given a word problem or a realistic task, we often hear students ask, "Do I multiply or divide?" Instead of persevering on their own, they rely on us, their teachers, to provide the appropriate arithmetic procedure. In some instances, they will simply multiply two numbers without justifying whether or not the answer makes sense in the context of the problem. Students often make errors in their computation that are compounded by a lack of understanding about the process they are using. As a result, we have found it necessary to re-engage students in multiplicative thinking. By doing so, we hope to deepen their knowledge of multiplication so that they can persist when encountering multiplication in new contexts throughout middle school and high school.

Multiplicative Sense Making

We teach with awareness of where students are in their current understanding of multiplication. At the same time, we look ahead to the mathematical learning that will connect to those understandings. The Common Core State Standards for Mathematics (CCSSM) contains numerous connections that will require fluency with multiplication beyond a memorized procedure (CCSSI 2010). As early as the sixth grade, students will need to expand their multiplicative thinking to the following:

1. Ratio and proportion (6.RP, p. 42)
2. Number system knowledge, including factoring and integer operations (6.NS, pp. 42-43)
3. Algebraic expressions and equations, in which students write expressions, identify equivalencies, and evaluate multiplicative expressions of real-world problems using algebraic notation (6.EE, pp. 43-44)
4. Geometry, in which students connect multiplicative ideas to areas of rectangles, triangles, special quadrilaterals, and volumes of prisms (6.G, pp. 44-45)

Once the full implementation of CCSSM occurs, we are hopeful that students will enter middle school with multiplicative fluency. That is to say, students' thinking about multiplication will be flexible, efficient, and precise. Such fluency can open learning spaces through which students can deepen their mathematical knowledge while making sense of CCSSM's middle school domains.

The way that CCSSM is organized in the lower grades allows this development to occur as students meet each grade's fluency mark through problem solving and modeling new mathematical ideas, early tenets of the Standards for Mathematical Practice. However, those of us working in middle schools should prepare to receive students who have not been exposed to the same set of coherent standards during their mathematics learning. In the next few years, students will most likely have some instruction implemented from CCSSM and from previous state and local standards. For these reasons, we plan to assess students' multiplicative sense making and re-engage them using activities like dot arrays to develop opportunities for future connections.

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Seeing Spots and Developing Multiplicative Sense Making Continued...

Dot Arrays

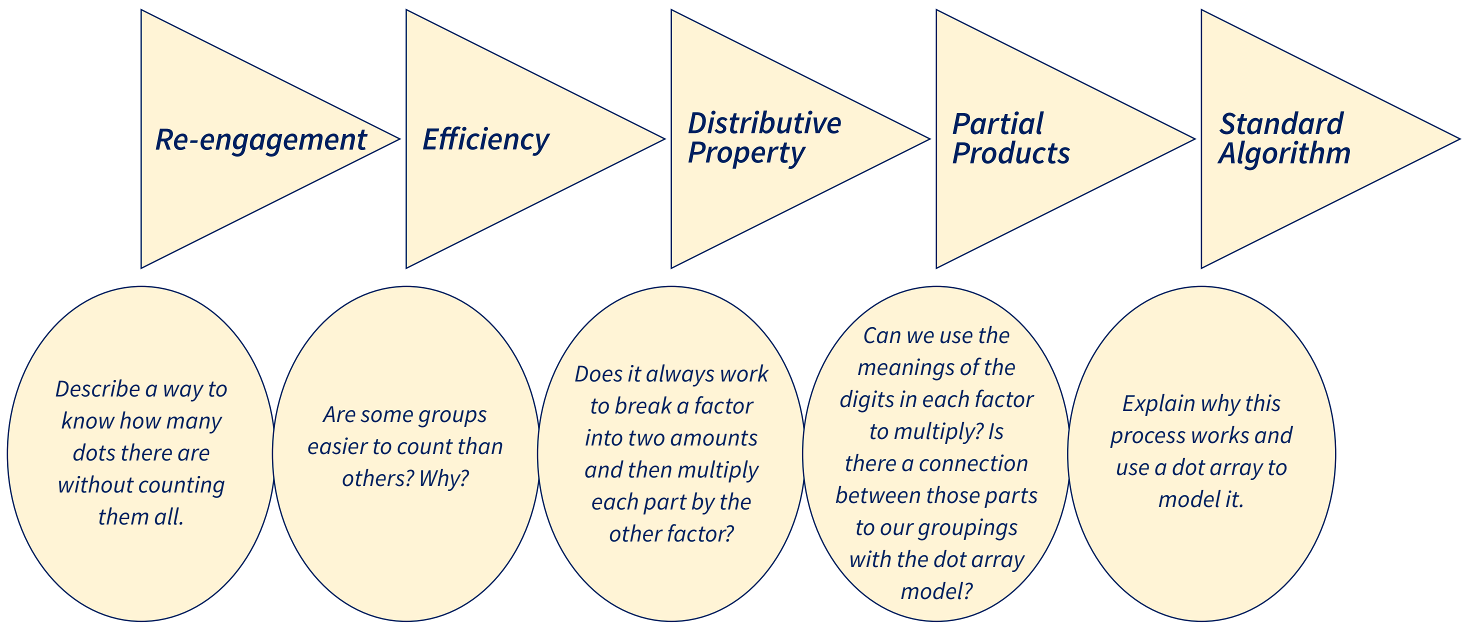
Our experience with individualized education plans has led us to develop dot array tasks that we have found beneficial for all students. These tasks allow students to reconnect and redevelop a sense for multiplicative efficiency and understanding, particularly with large numbers. They quickly come to realize that our expectations go beyond a memorized procedure and instead involve reasoning and justification. We begin this process with the dot array model because it allows for re-entry into multiplication by all students and promotes opportunities for creative thinking and various representations.

The structure of a dot array is particularly powerful because with each array, we are able to choose two particular grouping arrangements. We can then observe how students use (or do not use) the groups that are given. Furthermore, many students are somewhat familiar with arrays and their connection to multiplication, yet few have ever been given tasks with arrays larger than 6 × 6.

As the unit progresses, the sophistication of the tasks increases. Students also discover more efficient strategies with the array model and connect these strategies with more abstract forms of representation. The classroom discourse ebbs and flows among exploring efficiency and representation, analyzing the thinking of others, and explaining the meaning of multiplication. Figure 2 shows a typical stream of tasks and discussions that derive from students' work.

Fig. 2

This multiplicative re-engagement flow typifies a stream of tasks and discussions derived from the work of students.



Re-engagement into Multiplication

To begin, we give students dot arrays and ask them to describe some ways to ascertain the number of dots without counting them all. Students may make any decision that feels natural for them. We establish the expectation that no one can read their minds, so they must explain their strategy using drawings, words, or symbols. The openness of the task allows us to watch and consider students' thinking, highlight the differences in approaches through presentation of students' solutions, and orchestrate productive mathematics discourse (Smith and Stein 2011) about multiplication and its representations.

Figure 3 shows examples of dot array work. As students create and share their ideas, we are looking for examples that can both expand and deepen their mathematical views. In figure 3a, the student used an 8 × 13 array and explained, "Five groups of sixteen is easy, that's 80, so I just added the 24 and made 104." The idea that we can work out multiplication problems using a combination of addition with smaller multiplications is significant, as it moves beyond the idea of 8 × 13 as meaning only 8 groups of 13.

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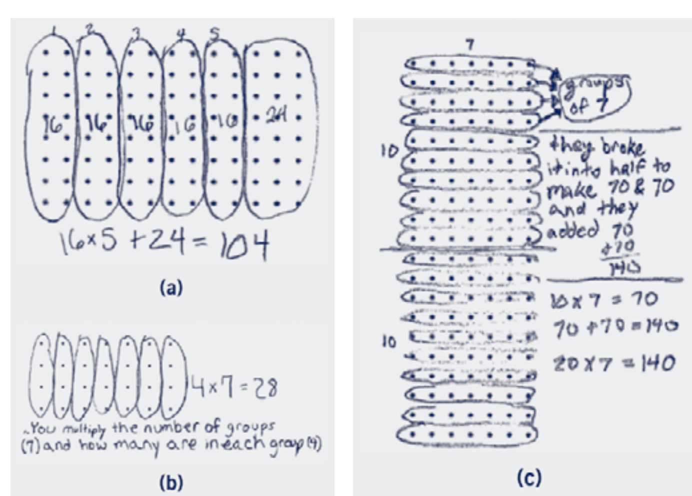
Fig. 3

Students approach in different ways the task of finding the number of dots without counting.



A solution like that in figure 3a can act as a catapult, causing students to think about other meanings for multiplication. After discussing solutions that use addition with smaller examples of multiplications, students gain flexibility that they can later transfer to the more abstract symbolic meaning involved in explaining algorithms.

The example in figure 3b can also elicit discussions about the need for precision. This student's written expression matches the drawing by making 7 groups of 4. However, the number sentence was written as 4 × 7, implying 4 groups of 7. Several students typically represent their written multiplications in a way that does not follow convention, but they are given the opportunity to explain the meaning and demonstrate visually how the expression connects with their dot array groupings. These cases provide fresh opportunities to engage students in a discussion about the meaning and conventional representation of multiplicative expressions.

An important question that we raise for consideration is "How can dot arrays be used to give a justification for the commutative property?" We allow time for students to discuss the issue and consider why it might be advantageous to think of 4 × 7 and 7 × 4 as different, even though the product is the same by the commutative property. Students demonstrate that each M × N array can be turned 90 degrees to make an N × M array without changing the original.

Two other important aspects that are generated by these dot array re-engagements involve understanding and critiquing the reasoning of others and moving toward efficiency. We allow students to work in pairs and bounce ideas off each other. When students believe that they have a finished solution, their ideas are tested by another pair, to see if they can discover the ideas and whether or not the drawings, words, and symbols make sense. The work in figure 3c contains the text "They broke it into half to make 70 & 70…" These words were added later by another pair of students who were checking if enough information was provided to deduce a strategy. Students can negotiate with one another about what needs to be done to accurately state the intended strategy.

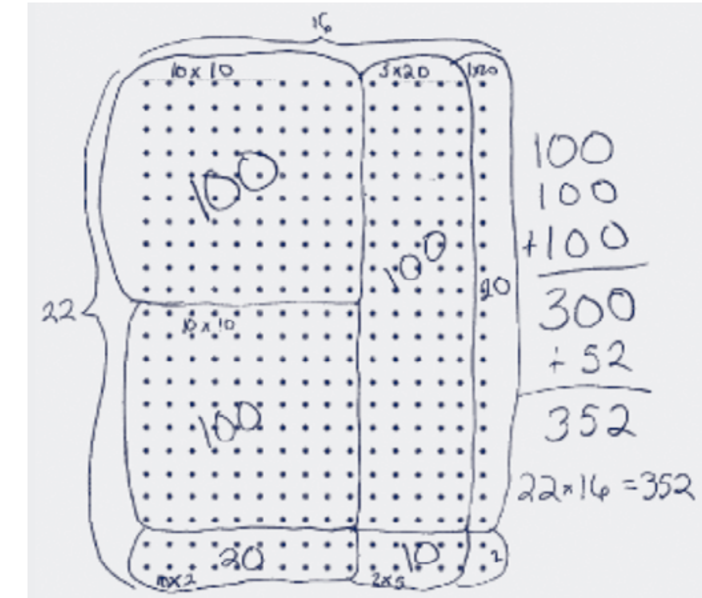
Another important aspect that we want to elevate through discourse is the careful choice of grouping that can produce an easier solution. Figure 3c was shared to promote this discussion. The idea of breaking 20 into 2 tens made it easier for students to find the answer of 20 × 7. The class decided that this strategy was more efficient than others. We highlight these examples as a natural transition to our discussions and expectations of increasing efficiency.

Efficiency

Students are given more dot arrays with the expectation of finding groups that relate to the base ten number system. They may organize these groups in whatever way makes the most sense to them, often choosing to find multiple groups of 10. The sample work of figure 4 shows that this student chose to make multiple groups of 10 or 20 to form larger groups of 100. Throughout their presentations and discussions, we encourage students to think about ways that they can improve their own efficiency.

Fig. 4

To find the product of 22 and 16, one student grouped the dots by 10s, trying to cluster 100 dots when possible.



In subsequent conversations, students often referred to others' ideas, such as "Joe's method," to indicate whose thoughts they were using to justify their new adaption. Although the approach for obtaining 22 × 16 used in figure 4 is more advanced in efficiency than others', connections can be made that will produce improvement. One such connection involves the move by some students to break up the main factors of the array to make more readily known multiplications over addition. When these cases occur, we discuss these strategies (see fig. 5) and their connection to the distributive property.

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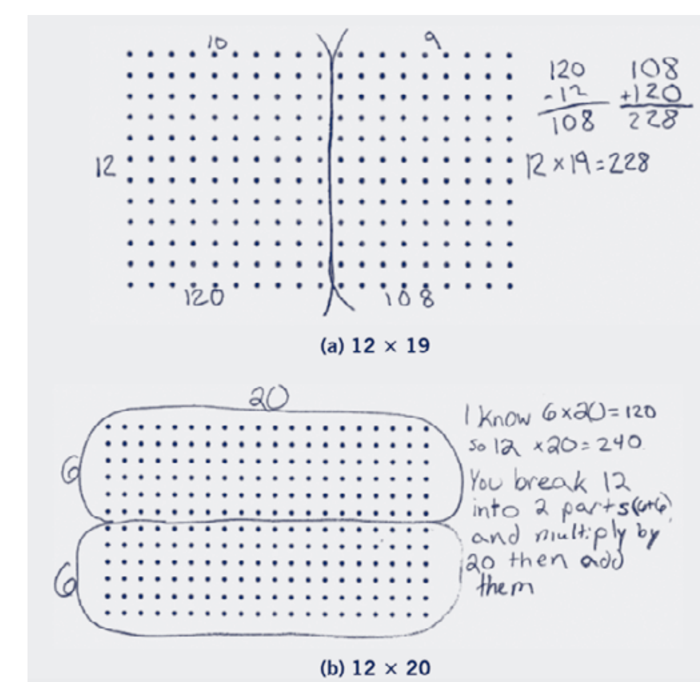
Fig. 5

The work of these students moved them toward thinking about the distributive property.



Distributive Property

Dot arrays can provide a clear picture of the distributive property because students can see how one factor is broken apart in ways that make multiplication by the other factor easier. In students' attempts to demonstrate a more efficient method using the dot array model, we invariably see work that exemplifies the distributive property. We look for the sort of student work shown in figure 5 to highlight—

- a concise representation of ideas;
- a decomposition of factors within any multiplication problem to make it easier to solve; and
- a consideration of whether or not both factors can be decomposed.

After being given time to think, students realize that the work in figure 5a and 5b, respectively, can be written concisely as the following:

$$12 \times 19 = 12 \times (10 + 9)$$
$$= 120 + 108$$
$$= 228$$

$$12 \times 20 = (6 + 6) \times 20$$
$$= 120 + 120$$
$$= 240$$

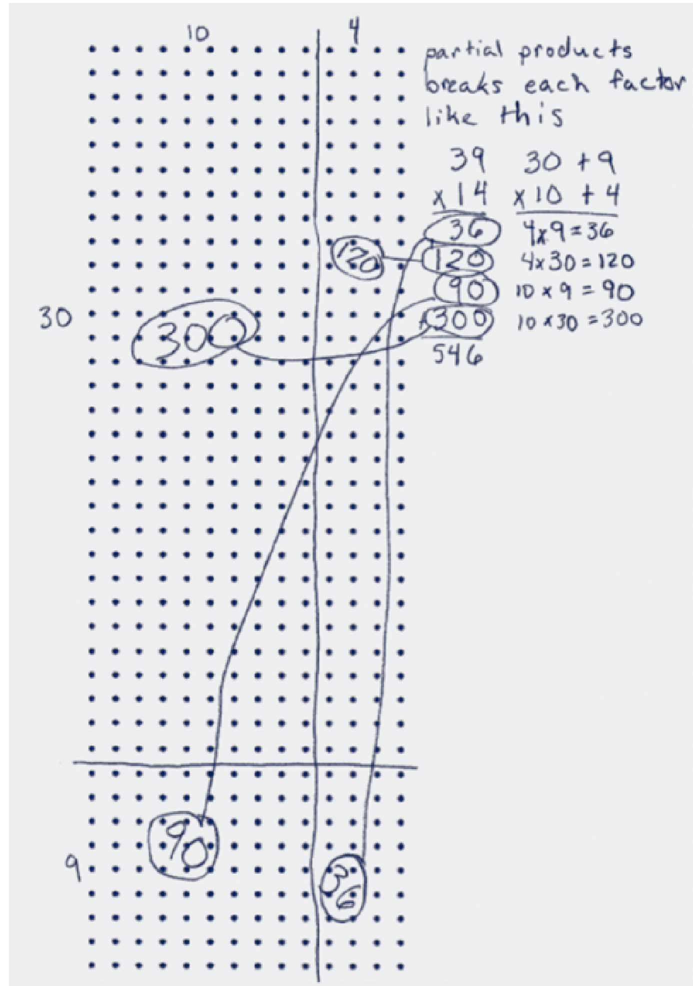
We ask students to work through other dot arrays and try this strategy because it opens a space for students to look for and make use of structure. It provides important experiences for their future consideration of $a(b + c) = ab + ac$. Looking beyond the trivial form of the distributive property, we ask students to consider what it would mean if we decomposed both factors into numbers of our choice and then multiplied. Often, students bring up this idea themselves as they notice nice groups that they have formed in their dot arrays. From these ponderings arise connections to partial products.

Partial Products

Some students' ideas gravitate naturally toward modeling the partial products algorithm (see figure 6) or toward dividing the dots into sections and finding the number of dots in each section. During our challenge to students to be more efficient, some of them discovered this strategy and decomposed both factors, although not always choosing the same decomposition. This situation provided the context for us to ask, "Can the multiplications in the partial products algorithm be represented by a dot array?" Students were initially allowed to explore this question with their choice of any size dot array. Many students worked through several cases and discussed their findings with others.

Fig. 6

The partial products algorithm was modeled in this dot array.



After students' ideas were shared, we asked them to demonstrate the partial products algorithm using a 39 × 14 array. The student work in figure 6 illustrates how each of the partial multiplications is geometrically parsed. We find that students benefit the most when we give them the task of connecting the place value meaning of the digits and the multiplicative process to the dot array model.

We often hear students make clarifying remarks about situations that we as teachers take for granted. During one class, a student exclaimed, "Oh! So that's why we add all of these numbers up after we multiply the pieces! I always thought that was weird, but yeah, they are just pieces of the whole." The dot arrays provide a visual model that can help students better understand the arithmetic processes, including the standard algorithm for multiplication.

Standard Algorithm

We conclude the re-engagement with multiplication by asking pairs of students to explain the process of the standard algorithm for multiplication using the dot array model. Students quickly see that the smaller products of the standard algorithm match those of the partial products algorithm. They also notice that the standard algorithm takes less writing because its representation allows two numbers to be summed in one step, as shown in figure 7.

We encourage students to be precise in their explanations and to ask questions of one another to account for all the details in the standard algorithm. "Why do we write the 3 from the 30 up there?" and "Why were we told to put a zero in the one's place on the second row?" are two of our favorite questions. After much discussion and presentation with their peers, each group writes an explanation and presents it to the class. The students actively seek to answer the questions that arise during discussion. We have found that students give richer explanations of the standard algorithm for multiplication when they have experiences with and use the dot array model.

Many students need the visual aspects to appreciate the intricacies and connections involved in mathematics. The dot array model allows students the freedom to explore mathematically appropriate representations of arrays and learn how to make more efficient decisions. Through these multiplication experiences, students can entertain mathematical questions like "In trying to find the product of 11 × 14, is it best to think of it as 11 groups of 14 and add 14 eleven times, or is there another more efficient way?"

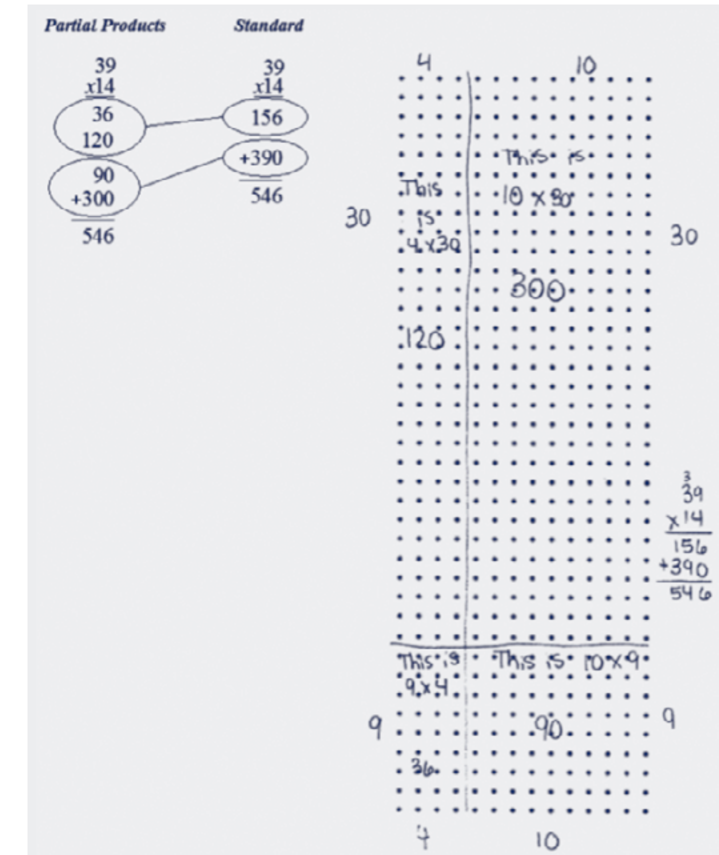
Ideas on the Horizon

These activities prepare students for making many of the mathematical connections they will encounter in their immediate future. For example, when we move into algebraically representing the distributive property, we invite students to think back to the dot arrays by asking, "How can we represent $12(x + 7)$?" We have also found that students who have significant experiences with dot arrays are much more adept at transitioning to similar models comprising more sophisticated forms of multiplication. For example, we have seen students use algebra tiles to think through multiplication of binomials. Student strategies also tend to transfer from the discrete nature of the dot array to the continuous nature of solving area problems. As they solve area problems, we have noticed an increase in their self-initiative in drawing auxiliary lines to break up areas into more manageable pieces.

We have described many ways that dot arrays can be used to deepen students' understanding of multiplication. Our school system has a transient population of students who are ESL and impoverished. Many of the older students who come to us do so with vast deficits in their knowledge. These same tasks were used by teachers at the high school level to help students make sense of multiplication and apply this knowledge in their algebra courses. We encourage middle school and high school teachers to present the dot array model to their students who struggle to make sense of multiplicative ideas.

Fig. 7

Through their experiences with dot arrays, students can see that the standard algorithm for multiplication is consistent with their work in partial products.



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