Indiana State Mathematics Contest 2015

Geometry

<u>Do not open this test booklet until you have been advised to do so by the test proctor.</u>

This test was prepared by faculty at **Indiana University Purdue University Columbus**

Next year's math contest date: Saturday, April 23, 2016

(1) Which of the following statements is true?

a) A line intersects a circle at most once.
b) The interior angles of a 7 sided polygon sum to 900 degrees.
c) The largest angle in any triangle cannot exceed 90 degrees.
d) Both a) and b)
e) None of the above.

(2) How many points are on the line $y = 1$ and a distance of 2 units from the point (2,3)?

a) Exactly 0 points
b) Exactly 1 points
c) Exactly 2 points
d) Exactly 3 points
e) None of the above

(3) Which of the following describes the points in the xy-plane that are a distance of 4 units from (1,2)?

a) $x^2 + \frac{y^2}{4} = 16$
b) $(x - 2)^2 + (y - 1)^2 = 4$
c) $y - 2 = 4(x - 1)$
d) $(x - 1)^2 + (y - 2)^2 = 16$
e) None of the above

(4) A piece of paper is rolled up to make a cylinder (without a top or bottom). The circumference is 6 in and the length is 10 in. What is the surface area of the cylinder?

a) $\frac{3}{\pi}$ square in.
b) $\frac{30}{\pi}$ square in.
c) 120π square in.
d) 60 square in.
e) None of the above.

(5) Suppose that a and b are positive constants. Further, assume that the vertices of a triangle are (a, b), $(2a, b)$, and $(2a, 3b)$. What is the length (in terms of a and/or b) of the longest side of the triangle?

a) $2b$
b) $\sqrt{a^2 + 4b^2}$
c) $\sqrt{a^2 + b^2}$
d) $\sqrt{4a^2 + 9b^2}$
e) None of the above

(6) Suppose that L is a fixed positive length. There is a square with perimeter L, a circle with circumference L, and an equilateral triangle with perimeter L. Which of the following are true?

   a) The circle with circumference L has an area of $\dfrac{L^2}{4\pi}$ square units.

   b) The equilateral triangle has the largest area of the three.

   c) The square has a diagonal of length $\dfrac{1}{4}L$ units.

   d) Both a) and b)

   e) None of the above.

(7) Find the value of x in degrees in the figure drawn below (not drawn to scale) that has congruent angles marked.
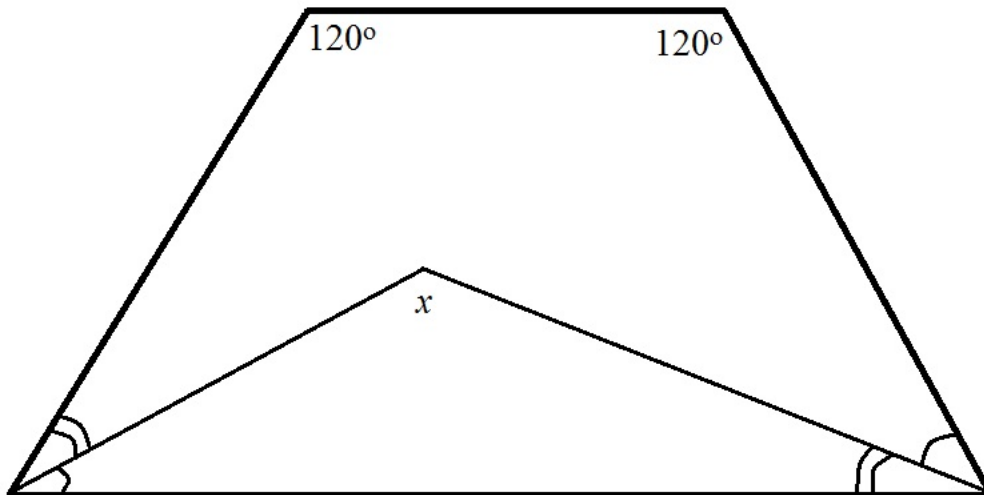
   a) 60 degrees
   b) 120 degrees
   c) 540 degrees
   d) 90 degrees
   e) None of the above

(8) Consider an equilateral triangle with sides of length 1 cm. Now use the midpoints of each side of the triangle to form a new equilateral triangle. Remove this new triangle from the original equilateral triangle to be left with 3 new equilateral triangles. Perform this process 3 more times as demonstrated by the diagram below. What is the total area of the remaining region (what is the area of the shaded region in the fifth step of the diagram)?
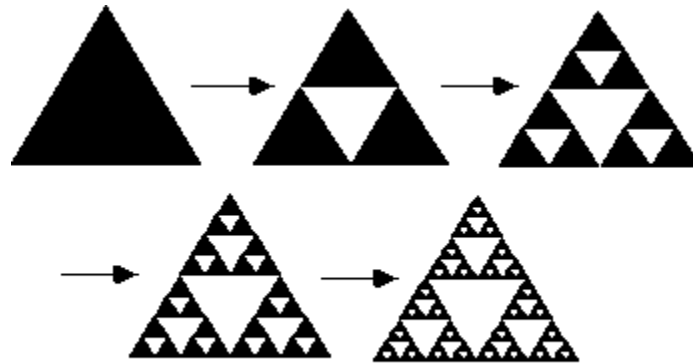
a) $\dfrac{\sqrt{3}}{4}$ square centimeters

b) $\dfrac{3}{4}$ square centimeters

c) $\dfrac{27\sqrt{3}}{64}$ square centimeters

d) $\dfrac{81\sqrt{3}}{1024}$ square centimeters

e) None of the above

(9) Below, a rectangle is divided into 4 smaller sub-rectangles by two perpendicular line segments. In square inches, the area of three of the smaller sub-rectangles is labeled. What is the sum of the four sub-rectangles?


a) 47 square cm
b) 59 square cm
c) 62 square cm
d) 56 square cm
e) None of the above

(10) Which of the following represents the points in the xy-plane that lie on a line parallel to the line segment connecting $(1,2)$ and $(4,6)$?

a) $y = \dfrac{3}{4}x + \dfrac{11}{4}$

b) $y = \dfrac{3}{4}x - \dfrac{1}{2}$

c) $4(x - 3) = 3(y - 2)$

d) $y = \dfrac{4}{3}$

e) None of the above

(11) Below is a circle of radius r centered at O. Further, suppose that A and B are two points on the circle and the triangle ΔOAB is equilateral. What is the area of the shaded segment?
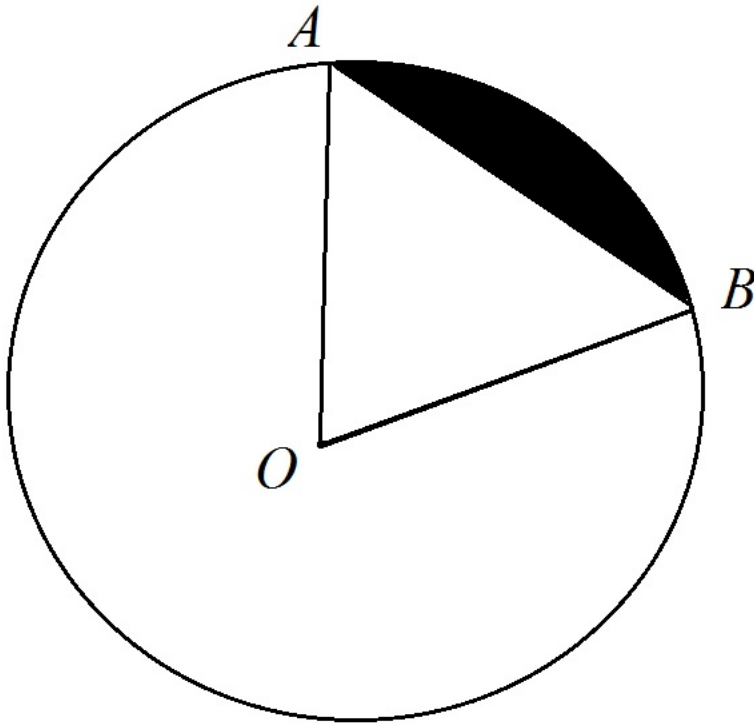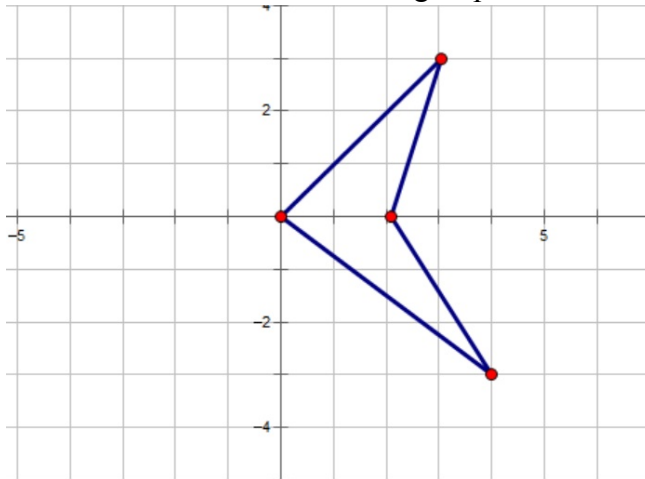
a) $\dfrac{\pi r^2}{6}$ square units

b) $\dfrac{\pi r^2}{6} - \dfrac{\sqrt{3}r^2}{4}$ square units

c) $\dfrac{\sqrt{3}r^2}{4}$ square units

d) πr^2 square units

e) None of the above

(12) Which of the following points in the *xy*-plane lie on the perpendicular bisector of the line segment connecting (2,3) and (1,2)?

    a) $\left(\frac{3}{2}, \frac{5}{2}\right)$
    b) (0,4)
    c) (5,3)
    d) Both a) and b)
    e) None of the above

(13) Find the area of the region enclosed by the quadrilateral below. You may assume everything is drawn to scale and that all vertices fall on grid points.

    a) 3 square units
    b) 6 square units
    c) 2 square units
    d) 4 square units
    e) None of the above

(14) Consider two cubes. If space diagonals (not face diagonals) of the second cube are twice the length of space diagonals for the first cube, the volume of the second cube is what factor of the volume of the first cube?

    a) A factor of 8
    b) A factor of 2
    c) A factor of $\frac{2}{\sqrt{3}}$
    d) A factor of $\sqrt{3}$
    e) None of the above

(15) You are told that the diagonal of a square is 1 unit. Which of the following are true?

a) Ignoring units, the area of the square is less than the diagonal of the square
b) The square can be inscribed inside a circle of area $\frac{\pi}{4}$ square units
c) The perimeter of the square is also 1 unit
d) Both a) and b)
e) None of the above

(16) What is the best answer to the following question? Is it possible for two planes to intersect at exactly one point?

a) No, this is not possible.
b) It is only possible if the planes are parallel.
c) It is only possible if the planes are perpendicular.
d) It is possible provided the planes meet at the origin.
e) None of the above.

(17) Consider the right triangle in the xy-plane with vertices $(0,0)$, $(100,0)$, and $(100,100)$. How many points (x, y), where both x and y are integers, either fall on the edges of the triangle or within the triangle?

a) 4500
b) 4950
c) 5050
d) 5151
e) None of the above

(18) Which of the following is NOT possible for a convex polyhedron?

a) Has exactly 21 vertices, 40 edges, and 21 faces
b) Has exactly 4 vertices, 6 edges, and 4 faces
c) Has exactly 8 vertices, 12 edges, and 6 faces
d) Has exactly 9 vertices, 16 edges, and 11 faces
e) None of the above

(19) Consider the line segment in the xy-plane connecting $(0,2)$ and $(2,0)$. What surface is generated when the line segment is revolved about the y-axis? (A third dimension is added when the line segment is revolved.)

a) A square triangle
b) A right circular cone
c) A cube
d) A cylinder of radius 2
e) None of the above

(20) Consider the trapezoid in the xy-plane with vertices $(0,0)$, $(1,1)$, $(2,1)$, and $(3,0)$. At what point in the xy-plane do the diagonals of the trapezoid intersect?

a) $\left(\dfrac{3}{2}, \dfrac{1}{2}\right)$

b) $\left(\dfrac{5}{2}, \dfrac{1}{2}\right)$

c) $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$

d) $\left(\dfrac{3}{2}, \dfrac{3}{4}\right)$

e) None of the above

(21) Consider a right triangle with a hypotenuse of length $\dfrac{3\sqrt{5}}{2}$ cm. Further, the other two sides of the triangle sum to 4.5 cm. What is the length of the second longest side of the triangle?
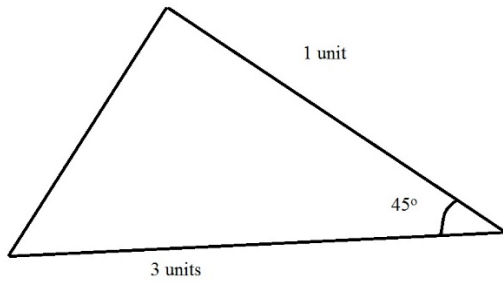
a) 2.25 cm

b) 3 cm

c) $\dfrac{3\sqrt{5}}{2}$ cm

d) 6 cm

e) None of the above

(22) What is the angle a in degrees (see the figure below)?
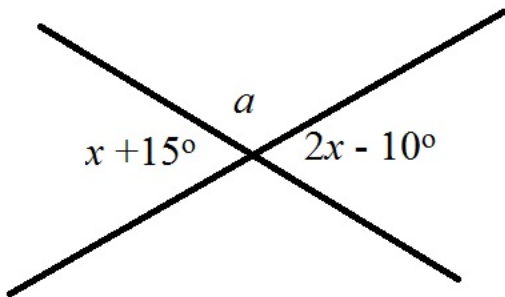
a) 40 degrees
b) 25 degrees
c) 120 degrees
d) 140 degrees
e) None of the above

(23) For the triangle below (may not be drawn to scale), what is the average of the three sides?
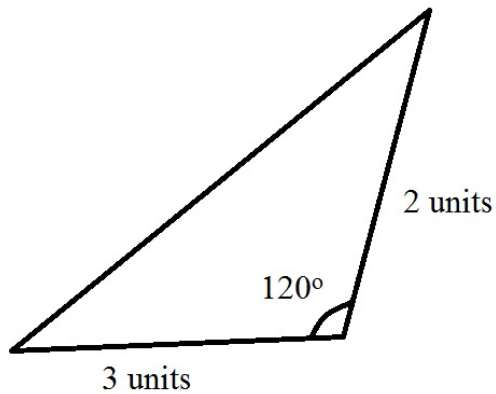

a) $\dfrac{4+\sqrt{10-\frac{6}{\sqrt{2}}}}{3}$ units

b) $\dfrac{3}{\sqrt{2}} - 1$ units

c) $\dfrac{4+\frac{3}{\sqrt{2}}}{3}$ units

d) $\dfrac{4+\frac{6}{\sqrt{2}}}{3}$ units

e) None of the above

(24)   For the triangle below (may not be drawn to scale), what is the area?

a) $6\sqrt{2}$ square units
b) $\sqrt{13}$ square units
c) $\dfrac{3\sqrt{3}}{2}$ square units
d) 6 square units
e) None of the above

(25) A chord which is a perpendicular bisector of a radius of length 4 in a circle, has length:

a) 8 units
b) $4\sqrt{3}$ units
c) $2\sqrt{3}$ units
d) 24 units
e) None of the above

(26) Below is a net for a right triangular prism with some of the edge lengths given. In cubic inches, how much volume does the right triangular prism hold?

5 in

4 in

3 in
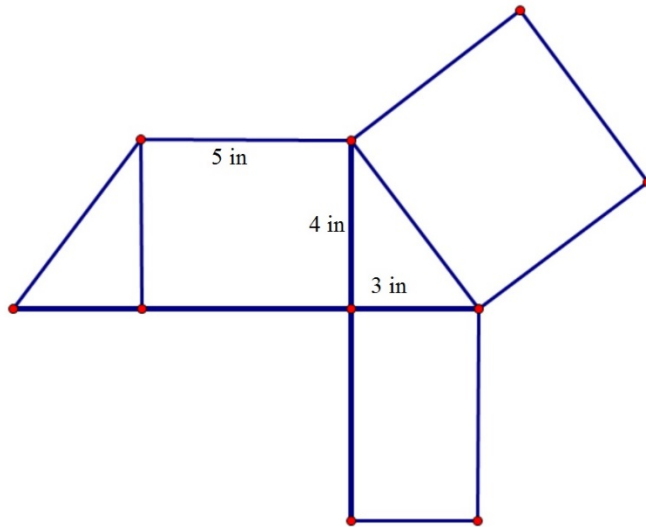
a) 20 cubic inches
b) 60 cubic inches
c) 30 cubic inches
d) 100 cubic inches
e) None of the above

(27) For an angle θ, $0^o < \theta < 90^o$, define the function

$$f(\theta) = (\cos(\theta) + \sin(\theta))(\cos(\theta) - \sin(\theta)).$$

Which value is $f(45^o)$?

a) 2
b) $\frac{\sqrt{2}}{2}$
c) $\sqrt{2}$
d) 0
e) None of the above

(28) Consider the trapezoid *ABCD* below (may not be drawn to scale). Assume that the line segments BC and AD are parallel. Provided that ∠A and ∠D are congruent, which of the following statements is always true?
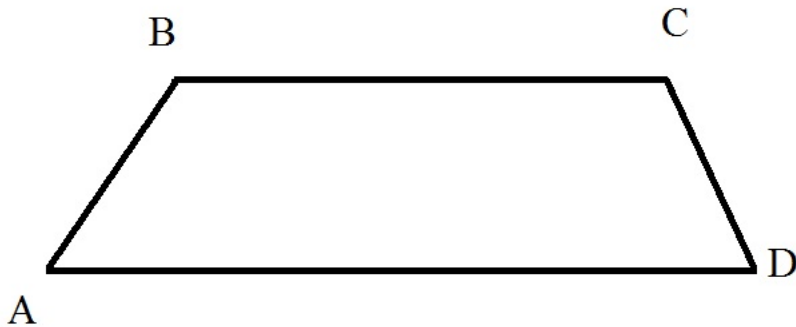
a) *ABCD* is an isosceles trapezoid
b) $\angle A = 60^o$
c) The sum of the four interior angles is 180°
d) Both a) and b)
e) None of the above

(29) For the following problem, find the value of x $(x = |CD|)$ in the following figure. Congruent angles and right angles are marked. The figure may not be drawn to scale.
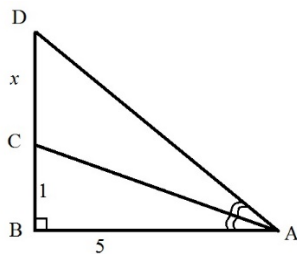

a) $\dfrac{13}{12}$ units

b) $\sqrt{26}$ units

c) $\dfrac{1}{\sqrt{26}}$ units

d) 1.25 units

e) None of the above

(30) Suppose that a point P is outside a circle and is a distance of 10 inches from the center. A secant from P cuts the circle at Q and R so that $|PR| = 15$ inches and $|PQ| = 7.5$ inches. What is the radius of the circle?

a) 5 inches
b) $5\sqrt{2}$ inches
c) $\dfrac{5}{4}$ inches
d) $\dfrac{25}{4}$ inches
e) None of the above

(31) An annulus is the region formed between two concentric circles (as shown in the shaded region in the figure below). If the radius of the smaller circle is 2, what should the radius of the larger circle be if the annulus is to have an area half that of the smaller circle?
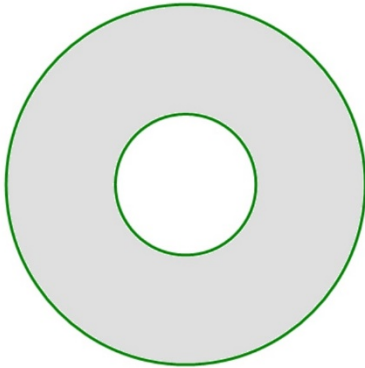
a) $\dfrac{3}{2}$ units
b) $\sqrt{6}$ units
c) $\sqrt{\dfrac{3}{2}}$ units
d) 4 units
e) None of the above.

(32) A fish tank (rectangular cuboid) is 3 feet by 4 feet by 2 feet. How much volume can the tank hold?

a) 12 cubic feet
b) 41,472 cubic inches
c) 24 cubic yards
d) 288 cubic inches
e) None of the above