Indiana State Math Contest
2018

Geometry

This test was prepared by faculty at Indiana University - Purdue University Columbus

Do not open this test booklet until you have been advised to do so by the test proctor.

1. The area of a certain right triangle is 210 cm^2. One of the legs of the triangle has length 20 cm. Find the perimeter of the triangle.

   (a) $40 + 20\sqrt{2}$ cm

   (b) $\frac{61+\sqrt{2041}}{2}$ cm

   (c) $20 + 10\sqrt{2}$ cm

   (d) 70 cm

   (e) None of the above.

2. The base angles of a trapezoid are each $30°$. The height of the trapezoid is 20 cm and the top base has length 10 cm. Find the area of the trapezoid.

   (a) 1000 cm^2

   (b) $400 + 200\sqrt{3}$ cm^2

   (c) $200 + 400\sqrt{3}$ cm^2

   (d) 500 cm^2

   (e) None of the above.

3. Each interior angle of a certain regular polygon measures $175°$. How many sides does the polygon have?

   (a) 48

   (b) 60

   (c) 180

   (d) 72

   (e) None of the above.

4. A right circular cone has a base with diameter of 20 mm and a volume of 75π mm^3. Find the total surface area (including base) of the cone.

   (a) $\frac{405\pi}{2}$ mm^2

   (b) 405π mm^2

   (c) $\frac{305\pi}{2}$ mm^2

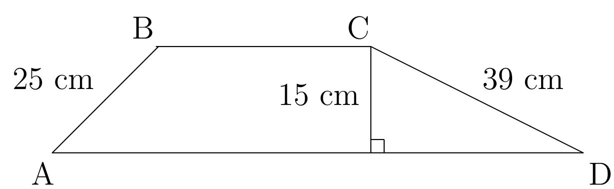
   (d) 305π mm^2

   (e) None of the above.

5. How many square centimeters are in one square kilometer?

   (a) 10^{10} cm^2
   (b) 10^8 cm^2
   (c) 10^{11} cm^2
   (d) 10^9 cm^2
   (e) None of the above.

6. The perimeter of the trapezoid $ABCD$ pictured below is 160 cm. The length of side AD is 76 cm. Find the area of trapezoid $ABCD$.



   (a) $120\sqrt{6}$ cm^2
   (b) $240\sqrt{2}$ cm^2
   (c) 480 cm^2
   (d) $240\sqrt{3}$ cm^2
   (e) 720 cm^2

7. The sum of the lengths of all the edges of a given cube is 180 in. Find the surface area of the cube.

   (a) 3375 in^2
   (b) 1350 in^2
   (c) 5400 in^2
   (d) 11390.625 in^2
   (e) None of the above.

8. Five distinct points, A, B, C, D, and E, lie on a line. We are given that

   - B is between E and C
   - C is between B and A
   - A is between C and D

   Which of the following can never be true?

   (a) B is between E and A

   (b) A is between E and C

   (c) C is between E and D

   (d) A is between B and D

   (e) C is between E and A

9. An isosceles triangle is inscribed in a semicircle of radius 1 in. Find the perimeter of the triangle.

   (a) $1 + 2\sqrt{2}$ in

   (b) 6 in

   (c) 3 in

   (d) $1 + \sqrt{2}$ in

   (e) $2 + 2\sqrt{2}$ in

10. Find the total number of diagonals of a polygon with 123 sides.

    (a) 7503

    (b) 7381

    (c) 7380

    (d) 7260

    (e) None of the above.

11. The lines with equations

$$2x + y = a, \quad x - 2y = b, \quad 3x + y = c$$

are all concurrent in the point $(2, -1)$. Which of the following is the value of $a+b+c$?

(a) 3

(b) -2

(c) -10

(d) 7

(e) None of the above.

12. The area of an equilateral triangle is numerically the same as its perimeter. What is the area of this triangle?

(a) $4\sqrt{3}$ units^2

(b) 6 units^2

(c) $12\sqrt{3}$ units^2

(d) $2\sqrt{3}$ units^2

(e) $3\sqrt{3}$ units^2

13. Find the area of a regular octagon each of whose sides has length 1 in.

(a) $4\sqrt{2}$ in^2

(b) $1 + 4\sqrt{2}$ in^2

(c) $2 + 2\sqrt{2}$ in^2

(d) $1 + \sqrt{2}$ in^2

(e) None of the above.

14. The area of a given rhombus is 3000 cm^2. One of its diagonals has length 50 cm. Find the perimeter of the rhombus.

(a) 480 cm

(b) 260 cm

(c) 240 cm

(d) $40\sqrt{61}$ cm

(e) None of the above.

15. Two angles of an isosceles triangle are x and $x + 42°$. Which of the following could NOT be one of the angles in this triangle?

    (a) $54°$
    (b) $74°$
    (c) $46°$
    (d) $88°$
    (e) $32°$

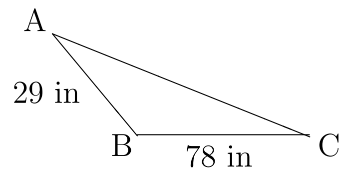
16. Which of the following statements is FALSE?

    (a) There is a quadrilateral with an interior angle of $300°$.
    (b) Every interior angle in a convex quadrilateral measures less than $180°$.
    (c) There is a quadrilateral whose interior angles are consecutive even integers.
    (d) The sum of the measures of any three interior angles in a quadrilateral must be less than $360°$.
    (e) All of the above.

17. In a certain triangle, the complement of the first angle is twice the supplement of the second. Also, the supplement of the first angle is twice the complement of the third. Find the measure of the third angle.

    (a) $22.5°$
    (b) $67.5°$
    (c) $45°$
    (d) $11.25°$
    (e) None of the above.

18. In the picture below, $\triangle ABC$ has area 780 in^2. Find the length of side AC.



(a) 121 in
(b) 101 in
(c) 97 in
(d) 113 in
(e) None of the above.

19. The lengths of the sides of a certain triangle are all consecutive odd positive integers. What is the smallest possible perimeter of such a triangle?

(a) 15
(b) 27
(c) 9
(d) 21
(e) 33

20. Chords $\overline{AB}$ and $\overline{CD}$ of a given circle intersect in a point P. We are given the following information:

- The length of $\overline{CP}$ is twice the length of $\overline{AP}$.
- The length of $\overline{PD}$ is 1 less than the length of $\overline{AP}$.
- The length of $\overline{PB}$ is 4 more than the length of $\overline{AP}$.

Find the length of the chord $\overline{AB}$.

(a) 16
(b) 17
(c) 20
(d) 12
(e) None of the above

21. A square and a semicircle each have the same perimeter. Find the ratio of the area of the square to the area of the semicircle.

    (a) $\frac{(\pi+2)^2}{16\pi}$
    (b) $\frac{(\pi+1)^2}{8\pi}$
    (c) $\frac{(\pi+1)^2}{16\pi}$
    (d) $\frac{(\pi+2)^2}{8\pi}$
    (e) None of the above.

22. Two of the legs of a right triangle measure 266 in and 312 in. Find the length of the median drawn from the vertex of the right angle to the hypotenuse.

    (a) 133 in
    (b) 410 in
    (c) 205 in
    (d) 156 in
    (e) 289 in

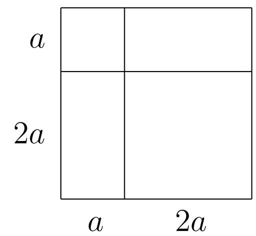
23. A piece of wire of length 1 ft is cut into two pieces. Each piece is bent to form a square. If the areas of the squares are in a ratio of $4:9$, find the perimeter of the square with the larger area.

    (a) $\frac{15}{17}$ ft
    (b) $\frac{2}{3}$ ft
    (c) $\frac{12}{13}$ ft
    (d) $\frac{3}{5}$ ft
    (e) None of the above.

24. The length, width, and height of a rectangular box are in the ratio $6:3:2$. If the length of a diagonal of the box is 42 in, find the surface area of the box.

    (a) 864 in^2
    (b) 1728 in^2
    (c) 2304 in^2
    (d) 2592 in^2
    (e) None of the above.

25. The large square below is partitioned by lines drawn parallel to the sides of the square. Note that there are two interior squares measure $a \times a$ and $2a \times 2a$.



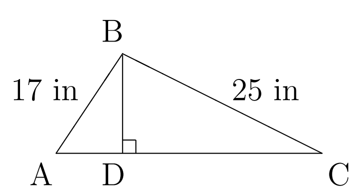
Four of these large squares will be placed next to each other (some rotated) to form a square that has dimensions $6a \times 6a$. Let N be the total number of squares in the resulting shape. What is the largest value of N?

(a) 18
(b) 16
(c) 10
(d) 14
(e) 22

26. Which of the following best describes the set of all points in three dimensions that are equidistant from three given noncollinear points?

(a) A point.
(b) A line.
(c) A plane.
(d) A cylinder.
(e) A sphere.

27. In the triangle below, the length of side AC is 28 in. Find the length of the altitude BD.



(a) $9\sqrt{3}$ in
(b) $12\sqrt{2}$ in
(c) 20 in
(d) $4\sqrt{34}$ in
(e) 15 in

28. A regular icosahedron is a solid polyhedron with 20 faces, all equilateral triangles. How many vertices does it have?

    (a) 18
    (b) 15
    (c) 16
    (d) 12
    (e) 20

29. The area of a certain triangle is 420 cm^2 and its perimeter is 98 cm. Find the radius of the triangle's inscribed circle.

    (a) $\frac{360}{49}$ cm
    (b) $\frac{480}{49}$ cm
    (c) $\frac{60}{7}$ cm
    (d) $\frac{21}{2}$ cm
    (e) 14 cm

30. A certain quadrilateral has two opposite right angles but is not a rectangle. The diagonal that divides this quadrilateral into two right triangles has length $\sqrt{65}$ in. If the lengths of all the sides of the quadrilateral are integers, what is the perimeter of this quadrilateral?

    (a) 20 in
    (b) 24 in
    (c) 30 in
    (d) 32 in
    (e) None of the above.

31. A regular tetrahedron is a solid polyhedron with 4 faces, all equilateral triangles. Suppose the midpoints of each edge of a tetrahedron T are joined to form an eight-sided figure, all of whose sides are equilateral triangles, called an octahedron. What is the ratio of the volume of the octahedron to the volume of the tetrahedron T?

(a) $\frac{1}{3}$

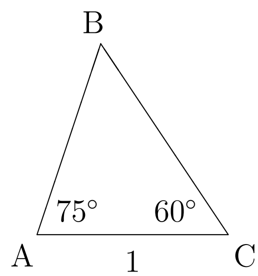
(b) $\frac{\sqrt{6}}{4}$

(c) $\frac{\sqrt{3}}{4}$

(d) $\frac{1}{2}$

(e) $\frac{2}{5}$

32. Find the length of side AB in the triangle below.



(a) $\frac{\sqrt{2}+\sqrt{6}}{2}$

(b) $\frac{\sqrt{6}}{2}$

(c) $\frac{\sqrt{6}-\sqrt{2}}{2}$

(d) $\frac{4\sqrt{3}}{3}$

(e) $\frac{\sqrt{3}+\sqrt{6}}{2}$