# *Indiana State Mathematics Contest*

2018

Algebra II/Integrated Math III

Do not open this test booklet until you have been advised to do so by the test proctor

This test was prepared by Dr. Paul Fonstad at **Franklin College**

1. Simplify the following expression: $\big((-2+5i)+(-1+3i)\big)\big((5)-(1-3i)\big)$.

    a. $12+41i$ b. $-36+23i$ c. $6+57i$ d. $28+29i$ e. $-6+17i$

2. Simplify the following expression: $\sqrt[10]{x^{15}} \times \dfrac{\sqrt[27]{x^{18}}}{\sqrt[12]{x^{10}}}$.

    a. $\sqrt[6]{x^5}$ b. $x\sqrt[3]{x}$ c. $x\sqrt[5]{x}$ d. $\dfrac{1}{\sqrt[6]{x^5}}$ e. $\sqrt[4]{x^3}$

3. If a and b are solutions to $\dfrac{4}{x-2}-\dfrac{3}{x+3}=1$, find ab.

    a. 24   b. -24 c. 30   d. Only one solution exists   e. No solutions exist

4. Simplify the expression: $\left(\dfrac{(x^2yz^0)^3}{x^4y^5}\right)^{-2}$.

    a. $\dfrac{y^4}{x^4}$ b. $\dfrac{xy}{z^3}$ c. x^2y^2 d. $-\dfrac{x^2}{y^2}$ e. $-\dfrac{x^4z^6}{y^4}$

5. What is the remainder when $3x^2-4x+8$ is divided by $x-2$?

    a. 28   b. -28 c. 12   d. -4 e. 20

6. Evaluate: $\sum_{k=7}^{12}(3k-2)$.

    a. 171   b. 181   c. 161   d. 159   e. 183

7. Which of the following are functions?

I.
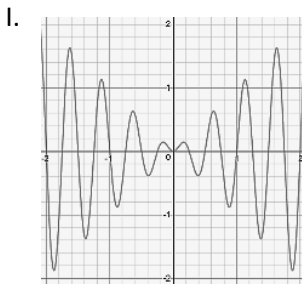
II. $\dfrac{3}{x^2+1}+4y=5$

III.

x	y
1	3
2	1
3	4
4	1

    a. I and II   b. I and III   c. I, II, and III   d. II and III   e. II

8. If $f(x) = x^2$, $g(x) = \frac{2}{x}$ and $h(x) = x + 3$, find $g\left(h(f(x))\right)$.

   a. $\frac{4}{x^2} + 3$ b. $\frac{2}{x^2} + 3$ c. $\frac{4}{(x+3)^2}$ d. $\left(\frac{2}{x} + 3\right)^2$ e. $\frac{2}{x^2+3}$

9. If $f^{-1}(x) = \frac{3x-2}{x+5}$, for what value(s) of x is $f(x)$ not defined?

   a. $\frac{5}{3}$ b. -2 c. $\frac{2}{3}$ d. -5 e. 3

10. If the graph of the function $y = 2 * 2^x - 2$ is reflected across the line $y = x$, which of the following points would <u>not</u> appear on the graph?

    a. $(6,2)$ b. $(-2,-1.5)$ c. $(2,1)$ d. $(0,0)$ e. $(-1,-1)$

11. What transformations must be done to $f(x) = x^5$ to turn it into $\left(\frac{x}{2}\right)^5 - 4$?

    a. Shrink the graph of $f(x)$ vertically by a factor of 2 and shift it right 4.
    b. Shrink the graph of $f(x)$ horizontally by a factor of 2 and shift it right 4.
    c. Stretch the graph of $f(x)$ vertically by a factor of 2 and shift it down 4.
    d. Stretch the graph of $f(x)$ horizontally by a factor of 2 and shift it down 4.
    e. None of the above.

12. Consider a circle with radius 5 centered at the origin and a line with a slope of 3 and a y-intercept of 5. If the intersection points of these two figures are (a,b) and (c,d), find $(ac + bd)$.

    a. 0 b. -2 c. -20 d. -12 e. These figures do not intersect at exactly two points.

13. If a, b, and c are numbers such that the system of linear equations below has infinitely many solutions, and if $c \neq 0$, find $\frac{a+b}{c}$.

    $$3x + 2y = 0$$
    $$ax + by = c$$

    a. 4 b. $-\frac{2}{3}$ c. $\frac{3}{2}$ d. $\frac{5}{8}$ e. No such values of a, b and c exist

14. If the point (a, b, c) is a solution to the system of linear equations below, find $a * b * c$.

    $$x + y + z = 6$$
    $$2x + 3y + 2z = 15$$
    $$3x + 4y + 4z = 23$$

    a. 0 b. 6 c. 72 d. -42 e. No such values of a, b and c exist

15. At the Historic Artcraft Theater in Franklin, tickets are sold as senior, child, or general admission. If a family buys 1 general ticket and 3 child tickets, it costs them $14. If another family buys 2 senior and 2 child tickets, it costs them $14. If a third family buys 1 senior and 2 general tickets, it also costs them $14. Suppose a family wants to buy 2 senior, 2 child, and 2 general tickets. How much will it cost them?

    a. $18   b. $22   c. $24   d. $28   e. $30

16. The height of a ball t seconds after it was kicked is given in the table below. Use this information to determine which quadratic equation most accurately models the ball height $h(t)$ at time t.

Time t (in seconds)	Height $h(t)$ (in feet)
0	0
1	24
2	16

    a. $h(t) = -16t^2 + 40t$
    b. $h(t) = -9.8t^2 + 33.8t$
    c. $h(t) = -32t^2 + 56t$
    d. $h(t) = -2t^2 + 26$
    e. $h(t) = -12t^2 + 36t$

17. If the equation $y = \frac{x^2}{2} - 2x + 7$ were rewritten in the form $y = a(x - h)^2 + k$ with appropriate values for $a, h,$ and k, which of the following expressions is largest?

    a. $k * h * a$ b. k^{h^a} c. $a - h + k$ d. $a + h - k$ e. $(k \div h) \div a$

18. For what value of a would the discriminant of $ax^2 + 6x - 4 = 0$ be equal to zero?

    a. 9   b. $-1\frac{1}{2}$ c. $1\frac{1}{2}$ d. $-2\frac{1}{4}$ e. $2\frac{1}{4}$

19. Which of the following <u>does not</u> describe the same sequence?

    a. A geometric sequence starting at 3 with a common ratio of 2.
    b. For his New Year's resolution, William decides to start with three push-ups on January 1st and do two more push-ups each day.
    c. An arithmetic sequence starting at 3 with a common difference of 2.
    d. $r_0 = 3, r_n = r_{n-1} + 2$
    e. $a_n = 3 + 2n$

20. Which of the following graphs has a y-intercept at 2, a horizontal asymptote of $y = 0$, and approaches positive infinity as x approaches positive infinity?

    a. $y = 2(3)^x$ b. $y = 2(3)^{-x}$ c. $y = -3(2)^x$ d. $y = -2\left(\frac{1}{3}\right)^x$ e. $y = 3(2)^x$

21. The function $P(t) = 500 * (.97)^t$ best describes which scenario?
   a. There are 500 velociraptors on Isla Nubar, and their population is growing by 97% a year.
   b. There are 97 troodons on Isla Nubar, and their population is growing by 500% a year.
   c. There are 500 pteranodons on Isla Nubar, and their population is growing by 97 a year.
   d. There are 500 dimetrodons on Isla Nubar, and their population is declining by 3% a year.
   e. There are 500 apatosauruses on Isla Nubar, and their population is declining by 97% a year.

22. At 8:00 am, Baljeet borrows $5 from Bufford the Bully, who charges 25% interest compound hourly. If Baljeet waits to pay Bufford back until 3pm that same day, how much money does he owe?
   a. $9.77   b. $5.03   c. $23.84   d. $37.46   e. $17.94

23. Simplify: $10^{\log(5)} + 3\log(2) + 2\log(5) - \frac{1}{2}\log(4)$.
   a. 5   b. 7   c. $5 + 6\log(5)$ d. $6 + \log(2)$ e. $30\log(10)$

24. Solve for x if $\log(x) + \log(x - 15) = 2$
   a. -5 b. 5   c. 5 and 20   d. 20   e. no solutions exist

25. The four walls of a rectangular room are to be painted. The length of the room is two feet more than the height, and the width of the room is 4 feet less than twice the height. If the total surface area of the four walls minus a 3 ft. by 7 ft. door is 331 square feet, what is the volume of the room?
   a. $960\ ft^3$ b. $784\ ft^3$ c. $630\ ft^3$ d. $498\ ft^3$ e. $384\ ft^3$

26. How many zeros does the following piecewise function have?
$$f(x) = \begin{cases} 4x - 2 & if\ x \leq 0 \\ x^3 - x^2 - 6x & if\ 0 < x < 4 \\ 2x^2 + 7x - 30 & if\ x \geq 4 \end{cases}$$
   a. 0   b. 1   c. 2   d. 3   e. 4

27. For a certain equation, x varies jointly with y^2 and inversely with z, and if $y = 8$ and $z = 4$, then $x = 8$. What must y be if $x = 2$ and $z = 9$?
   a. 8   b. 36   c. 24   d. 4   e. none of these

28. The number of cricket chirps varies linearly with the room temperature. If the room temperature is 60° F, the cricket will chirp 88 times per minute. If the room temperature is 57° F, the cricket will chirp 82 times per minute. If $T(c)$ is the temperature for c chirps per minute, which of the following is an equation for $T(c)$?
   a. $T(c) = \frac{1}{2}c + 16$ b. $T(c) = 2c - 32$ c. $T(c) = \frac{1}{3}c + 40$ d. $T(c) = \frac{5}{9}c + 32$ e. $T(c) = 2c + 12$

29. The mean of a data set with eight values is 80. If one of the data points is removed, the mean of the remaining data set becomes 84. What is the value of the data point that was removed?
   a. 48   b. 52   c. 56   d. 64   e. none of these

30. A jar contains 4 black and 6 red beads. A second jar contains 5 black and an unknown number of red beads. If, when you draw one bead from each jar the probability that you draw two beads of the same color is .55, how many red beads are in the second jar?

    a. 8   b. 10   c. 12   d. 15   e. 20

31. In how many different ways can you rearrange the letters in the word ALGEBRA ?

    a. 5040   b. 2520   c. 1260   d. 840   e. 420

32. Simplify: $\dfrac{\frac{1}{x+1}+\frac{1}{x-1}}{\frac{1}{x+1}-\frac{1}{x-1}}$.

    a. $1-x$ b. -1 c. x d. 1 e. $-x$