Indiana State Math Contest

2018

Algebra I

This test was prepared by faculty at the University of Southern Indiana

<u>Do not open this test booklet until you have been advised to do so by the test proctor.</u>

1. Solve $0.06x + 0.09(15 - x) = 0.07(15)$

a. 10 b. $\frac{15}{47}$ c. 3 d. -10 e. No solution

---

2. Solve $\frac{x+7}{6} + \frac{2x-8}{2} = -4$

a. -14 b. -1 c. $\frac{15}{7}$ d. $\frac{13}{7}$ e. 6

---

3. Solve $4(2x + 7) = 2x + 25 + 3(2x + 1)$

a. 0 b. 3 c. 14 d. All real numbers e. No solution

---

4. Solve $-3(x + 4) + 2 \geq 7 - x$

a. $x \geq -\frac{17}{2}$ b. $x \leq \frac{17}{2}$ c. $x \leq -\frac{17}{2}$ d. $x \leq 3$ e. $x \geq 3$

---

5. Solve $-\frac{2}{3}(x - 3) - \frac{1}{2} < \frac{1}{2}(5 - x)$

a. $x > 6$ b. $x > -6$ c. $x < -6$ d. $x < 6$ e. No solution

---

6. If $f(x)$ is a linear function with $f\left(\frac{1}{2}\right) = \frac{2}{3}$ and $f\left(\frac{2}{3}\right) = \frac{3}{4}$, what does $f\left(\frac{3}{4}\right) = $?

a. $\frac{6}{7}$ b. $\frac{19}{24}$ c. $\frac{6}{8}$ d. $\frac{4}{5}$ e. $\frac{7}{9}$

---

7. If x is nonzero, $\frac{x}{2} = y^2$, and $\frac{x}{4} = 4y$, then x is...

a. 8 b. 16 c. 32 d. 64 e. 128

---

8. Which line is perpendicular to $= 7$?

a. $x + y = 7$ b. $x - y = 7$ c. $y = -7$ d. $x = 7$ e. None of these

---

9. For real numbers $a, b, c,$ and d, $a - 1 = b + 2 = c - 3 = d + 4$.
   Which number is greatest?

a. a b. b c. c d. d e. Cannot be determined

---

10. If the price of a stock increased by 32% in 2015 then decreased by 25% in 2016, what was the overall effect on the price?

a. -7% b. -1% c. $+7\%$ d. $+8\%$ e. Cannot be determined

11. Harold has three children. The sum of the ages of the two younger children is the age of the oldest plus nine years. The sum of the ages of the two older children is the age of the youngest plus thirteen years. How old is the middle child?

a. 9   b. 10   c. 11   d. 12   e. 13

---

12. If $a + b = 1$ and $ab = -\frac{1}{2}$, then $a^2 + b^2 = ?$

a. 0   b. $\frac{1}{2}$ c. 1   d. $\frac{3}{2}$ e. 2

---

13. In a survey of US households, 85% own a working car, 26% a working motorcycle, and 6% own neither. What percent of car owners do not own a motorcycle?

a. 59   b. 65   c. 68   d. 80   e. 95

---

14. After years of experience, a math teacher finds that when grading 45 papers there are usually 3 with no name on them. If the teacher grades 240 papers in a week, how many should she expect to find with no name?

a. 5   b. 13   c. 16   d. 65   e. 80

---

15. The world population of red-crowned cranes is estimated at 2000. The chart shows how the population is divided among the four countries where they are found. About how many cranes live outside of North and South Korea?

Red-Crowned Crane Population

South Korea

North Korea

Japan

China

14%
40%
16%
30%
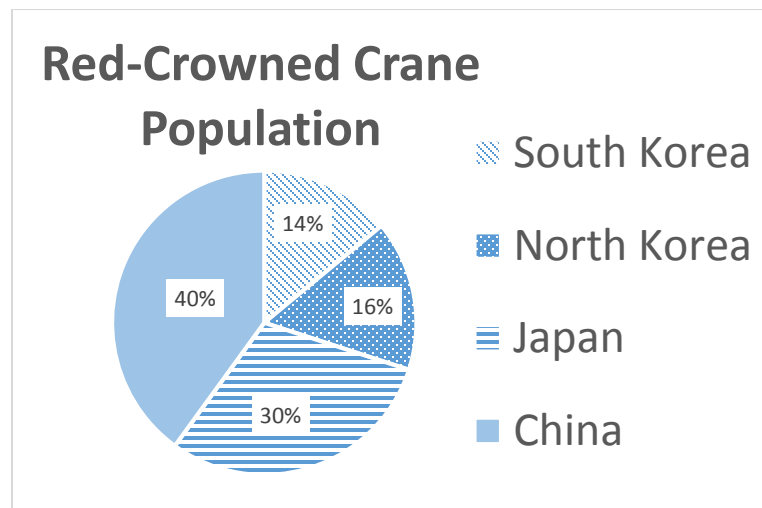
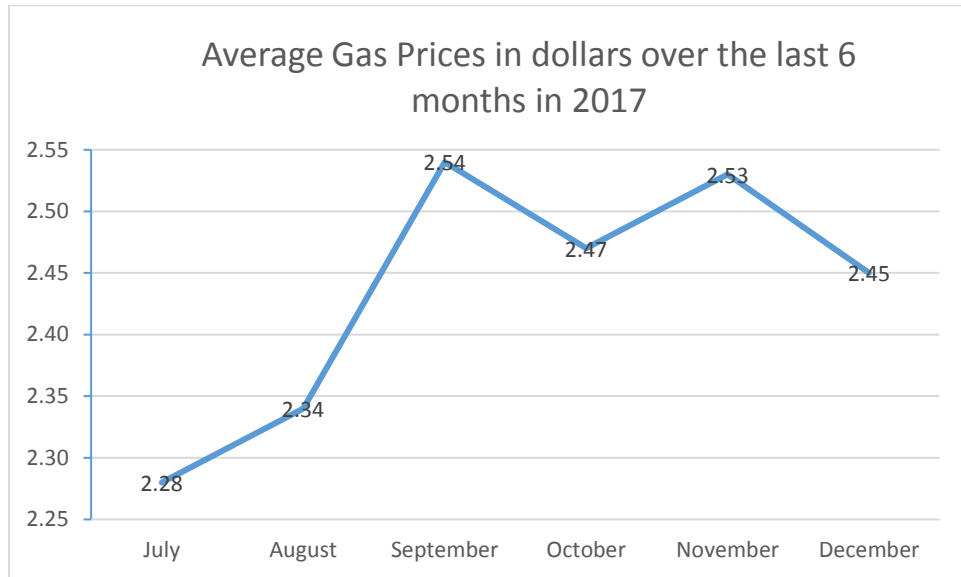a. 280   b. 320   c. 600   d. 800   e. 1400

---

16. If one solution of $5x^2 - 12x = b$ is $x = 3$, what is the other solution?

a. -3.6 b. -0.6 c. 6   d. 9   e. None of these

17. The graph shows the average gas price in the US for the last six months of 2017. How is this graph misleading?

Average Gas Prices in dollars over the last 6 months in 2017

a. The unequal vertical scale exaggerates the monthly price differences.

b. The vertical scale starting at $2.25 exaggerates the monthly price differences.

c. The lack of numbers on the horizontal axis gives no price information.

d. The graph only covers six months which is not enough time to analyze price changes.

e. The labels on the line make it difficult to read.

---

18. A rock is thrown so that its height t seconds after release is given by the function $h(t) = -16t^2 + 80t + 4$. How many of the following statements are true?

    I.   The rock is in the air for exactly 4 seconds
    II.   The rock reaches a maximum height of 80 feet
    III.   The rock reaches its maximum height at 2.5 seconds

a. I only   b. II only   c. III only   d. All of them   e. None of them

---

19. If 4 and 6 are solutions to $ax^2 + bx + c = 0$, what are solutions to $ax^2 - bx - c = 0$?

a. $\{-2,12\}$ b. $\{-4, -6\}$ c. $\{-12,2\}$ d. $\{-12, -2\}$ e. $\{-4,6\}$

---

20. Determine all values for c which the equation $3x^2 + 18x + c = 0$ has no real solutions.

a. $c > 0$ b. $c > 6$ c. $c > 18$ d. $c > 21$ e. $c > 27$

---

21. When a polynomial, $P(x)$, is divided by $x - 3$ the quotient is $2x^2 - 4x + 3$ and the remainder is 2. What is $P(2)$?

a. -1 b. 1   c. 3   d. 5   e. None of these

---

22. If $y = a(x - 2)^2 + c$ and $y = (2x - 5)(x - b)$ represent the same quadratic equation, what is the value of b?

a. 3   b. $\frac{3}{2}$ c. $\frac{4}{5}$ d. $-\frac{5}{2}$ e. $\frac{8}{5}$

---

23. The sum of two integers is 24. Seven times the smaller is two less than three times the larger. What is the value of the larger?

a. 7   b. 9   c. 15   d. 17   e. 20

---

24. The sum of the squares of two consecutive positive odd integers is 130. What is the value of the smaller integer?

a. 3   b. 5   c. 7   d. 9   e. 11

---

25. Pump A takes 16 minutes longer to empty a tank than pump B. The two of them working together can empty the tank in 6 minutes. How long would it take if only pump B were emptying the tank?

a. 8   b. 12   c. 13   d. 24   e. 26

---

26. The maximum load of a beam varies directly with its width and inversely with the square of its length. If a beam 2 inches wide and 12 feet long can hold a maximum of 1200 lbs, what is the maximum load of an 8 foot beam that is 4 inches wide?

a. 1600   b. 3600   c. 5400   d. 8640   e. 16000

---

27. A mixture of nuts and candy sells for $1.90 per pound. If the nuts sell for $2.25 per pound and the candy $1.50 per pound, how many pounds of nuts should be used to make a 60-pound mixture?

a. 20   b. 25   c. 28   d. 30   e. 32

---

28. The average price of oranges, apples, and limes is 50 cents per pound. Six pounds of oranges, five pounds of apples, and two pounds of limes cost $6. If limes are 25 cents per pound more than apples, what is the price of a pound of oranges?

a. 40¢   b. 45¢   c. 50¢   d. 55¢   e. 60¢

29. A spherical stone of diameter 4 feet is dropped into a rectangular tank that is 6 feet by 8 feet. If the stone is completely submerged, how much does the water level rise, to the nearest tenth of a foot?

a.   0.3 feet   b. 0.7 feet   c. 1.8 feet   d. 3.2 feet   e. 8.4 feet

---

30. Find the solution to $\begin{cases} 3x + 5y = 7 \\ 2x + 8 = 6y \end{cases}$

a. $\left(\frac{1}{4}, \frac{5}{4}\right)$ b. $\left(\frac{1}{14}, \frac{19}{14}\right)$ c. $\left(\frac{13}{7}, \frac{2}{7}\right)$ d. $\left(\frac{1}{29}, \frac{40}{29}\right)$ e. $\left(\frac{41}{14}, -\frac{5}{14}\right)$

---

31. Over the course of an ultra-marathon, Kim ran 3 kph faster than Susan. If Kim can run 15 km in the same time that Susan runs 9 km, what is Susan's speed?

a. 4.5 kph   b. 5 kph   c. 5.5 kph   d. 12 kph   e. 15 kph

---

32. Joel is selling custom cell phone cases for $10 and dog treats for $4. He needs to sell $1200 worth of items to afford a trip. If he knows he will sell between 100 and 200 dog treats, what is the least number of cell phone cases he will need to sell?

a. 0   b. 20   c. 40   d. 80   e. 120

---

33. Which of the following is not a function?

a.

x	-2	-1	0	1	2
$f(x)$	1	0	1	4	9

b.

x	-2	-1	0	1	2
$f(x)$	3	4	5	6	7

c.

x	-2	-1	0	1	2
$f(x)$	0	0	0	0	0

d.

x	-2	-1	0	1	2
$f(x)$	-2	-1	0	1	2

e.   They are all functions

---

34. Assume a linear relationship between number of visitors to a website and the charge for an ad on the site. If a page getting 4.2 million hits a month charges $98 for an ad, how much should they charge if their traffic increases by 1.2 million?

a. $28   b. $99.20   c. $101   d. $126   e. $343

35. If $f(x) = x^2$, which of the following is the table for $g(x) = f(x + 1)$?

a.

x	-1	0	1	2	3
$g(x)$	4	1	0	1	4

b.

x	-2	-1	0	1	2
$g(x)$	5	2	1	2	5

c.

x	-3	-2	-1	0	1
$g(x)$	4	1	0	1	4

d.

x	-2	-1	0	1	2
$g(x)$	3	0	-1	0	3

e.

x	-2	-1	0	1	2
$g(x)$	4	1	0	1	4

36. Over which interval is $f(x) = -x^2 + 6x - 8$ decreasing?

a. $(-\infty, \infty)$ b. $(3,1)$ c. $(3, \infty)$ d. $(-\infty, -8)$ e. $(-8,6)$

37. What is the domain of $f(x) = \frac{\sqrt{-x}}{(3+x)(3-x)}$?

a. $(-\infty, -3) \cup (-3, 0]$ b. $[0,3) \cup (3, \infty)$ c. $(-3,3)$ d. $(-\infty, \infty)$ e. $\emptyset$

38. If $h(x) = (f \circ g)(x) = \sqrt{8 - 3x}$, which of the following could be true?

    I. $f(x) = \sqrt{x}$ and $g(x) = 8 - 3x$
    II. $f(x) = x^2$ and $g(x) = \frac{8-x}{3}$
    III. $f(x) = \sqrt{8 - x}$ and $g(x) = 3x$

a. I only   b. II only   c. III only   d. I and III only   e. All of them

39. If $x \text{ ⋔ } y = 2x + y^2$, solve $(x - 1) \text{ ⋔ } (x - 2) = 3x - 4$.

a. $\{1,2\}$ b. $\{2,3\}$ c. $\{3,4\}$ d. $\{4,5\}$ e. $\{5,6\}$

40. What is the sum of the four solutions to the equation $(x^2 - 8x - 1)^2 - 64 = 0$?

a. 0   b. 8   c. 16   d. 64   e. -72