Indiana State Mathematics Contest 2019

Geometry

Do not open this test booklet until you have been advised to do so by the test proctor

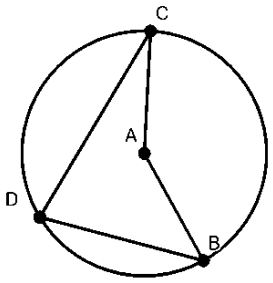
This test was prepared by Dr. Paul Fonstad at **Franklin College**

ICTM Geometry 2019

1. Determine the distance from the point (2,6,9) to the origin.
   a. 11 units
   b. 17 units
   c. 13 units
   d. 9 units
   e. None of the above

2. For the circle centered at A pictured below, if the measure of $\angle CDB$ is 87°, what is the measure of $\angle CAB$?

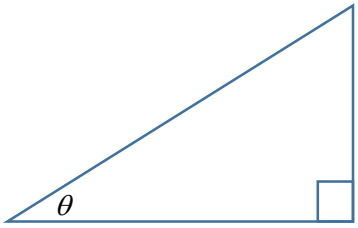
   a. 164°
   b. 174°
   c. 154°
   d. Cannot be determined
   e. None of the above



3. A waffle cone has a height of 6 inches and a diameter of 2 inches at its opening. If 1 teaspoon of batter makes 1 square inch of waffle, to the nearest teaspoon, how many teaspoons of batter are needed to make the cone?
   a. 52 teaspoons
   b. 22 teaspoons
   c. 40 teaspoons
   d. 19 teaspoons
   e. None of the above

4. A certain three-dimensional solid has 10 faces and 20 edges. How many vertices does it have?
   a. 8
   b. 22
   c. 18
   d. 32
   e. None of the above

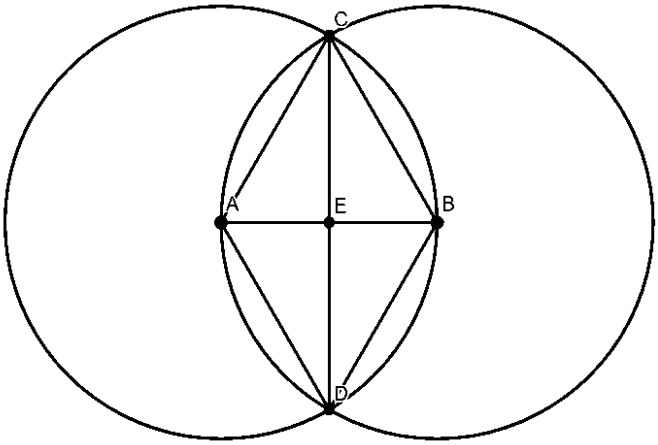
5. For the triangle below, as the angle θ changes from 0° to 90°, which of the four values listed below will never be the largest?



   a. $\cos\theta$
   b. $\sin\theta$
   c. $\tan\theta$
   d. 0.95
   e. Cannot be determined

6. If nylon is \$21 per **square yard**, how much would the fabric cost to make a triangular sail with a base of 12 **feet** and a height of 24 **feet**?
    a. \$3,024
    b. \$336
    c. \$672
    d. \$2,016
    e. None of the above

7. Based on the geometric construction below of two circles centered at *A* and *B*, where E is the intersection of $\overline{AB}$ and $\overline{CD}$, which of the following statements is **false**?



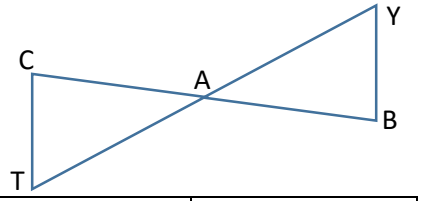
    a. $\angle ADB$ is a 60° angle
    b. E is the midpoint of $\overline{AB}$
    c. $\overline{CD}$ is perpendicular to $\overline{AB}$
    d. ΔABC is an equilateral triangle
    e. None of the above

8. To hold their cooking spices, Kelly, Darbie, and Hannah want an open top box with a volume of 216 cubic inches. Which of the following box options uses the least amount cardboard to make the box? (measurements are length by width by height in inches).
    a. 6 by 6 by 6
    b. 4 by 6 by 9
    c. 12 by 18 by 1
    d. 8 by 9 by 3
    e. 9 by 12 by 2

9. If the radius and height of a cylinder are both doubled, by how much will the volume grow?
    a. 8 times the original volume
    b. 6 times the original volume
    c. 4 times the original volume
    d. 2 times the original volume
    e. None of the above

10. For the two column proof below, fill in the missing line.

Given that $\overline{CA} \cong \overline{BA}$ *and* $\overline{TA} \cong \overline{YA}$, *prove that* $\Delta CAT \cong \Delta BAY$



$\overline{CA} \cong \overline{BA}$	Given
$\overline{TA} \cong \overline{YA}$	Given
????	????
$\Delta CAT \cong \Delta BAY$	SAS congruency

a. $\angle ACT \cong \angle AYB$ | Alternate interior angles
b. $\angle CAT \cong \angle YAB$ | Corresponding angle theorem
c. $\angle CAT \cong \angle YAB$ | Vertical angle theorem
d. $\angle ACT \cong \angle AYB$ | Similar triangles
e. None of the above

11. A barrel-maker is asked to construct an iron band to go around the center of a wooden barrel, with a 6-inch overlap. A short time later, he is asked to construct another iron band to go around another barrel whose radius is exactly 1 foot bigger than the first, with the same 6-inch overlap. To the nearest tenth of a foot, how much longer will his second iron band need to be?
    a. 6.3 feet
    b. 3.1 feet
    c. 1.6 feet
    d. Cannot be determined
    e. None of the above

12. Which of the following triples **cannot** be the side lengths of a triangle?
    a. 3, 4, 5
    b. 10, 12, 13
    c. 32, 255, 257
    d. 7, 7, 7
    e. All of the above could be the side lengths of a triangle.

13. Let ΔNEW be similar to ΔOLD. If the length of $\overline{OD}$ is four times the length of $\overline{NW}$, and the area of ΔNEW is 24 ft^2, what is the area of ΔOLD?
    a. 96 ft^2
    b. 192 ft^2
    c. 384 ft^2
    d. Cannot be determined
    e. None of the above

14. Which of the following statements are true?

I. If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle.

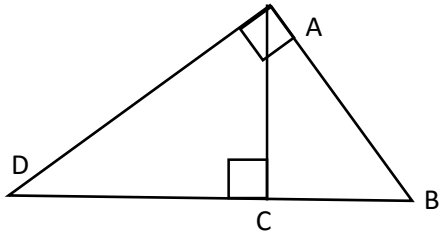
II. If a quadrilateral is a rectangle, then the diagonals of the quadrilateral are congruent.

III. The diagonals of a quadrilateral are congruent if and only if the quadrilateral is a rectangle.

a. Statement I only
b. Statement II only
c. Statement III only
d. Statements I, II, and III
e. None of the statements are true

15. Assume line l passes through the points (4,1) and (1,5). If line m is perpendicular to l and passes through the point (1,1), what other point must line m pass through?
    a. (-3, 4)
    b. (-2,4)
    c. (3,4)
    d. (5,4)
    e. None of the above

16. In the figure below, if the length of $\overline{CD}$ is 9 and the length of $\overline{BC}$ is 4, determine the length of $\overline{AC}$.



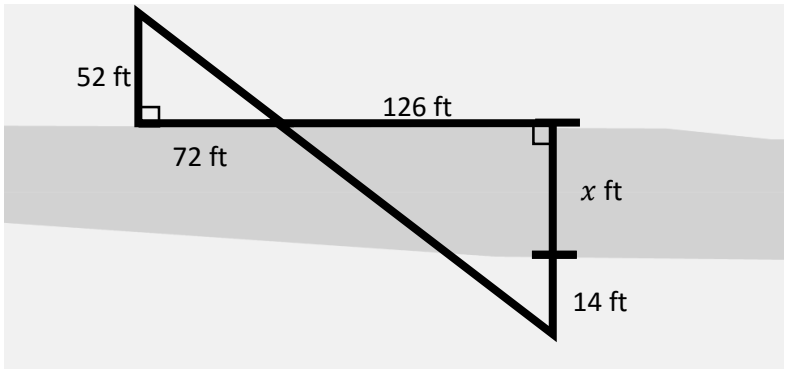
    a. 6
    b. $3\sqrt{2}$
    c. 5
    d. $2\sqrt{3}$
    e. None of the above

17. On the xy-plane, if a is a positive constant, find the length of the line segment between the points $(a, 18a)$ and $(9a, 3a)$.
    a. $9a\sqrt{3}$
    b. $15a$
    c. $18a$
    d. $21a$
    e. None of the above

18. Given ΔCAT and ΔDOG, which of the following **would not** be sufficient to prove that $\Delta CAT \cong \Delta DOG$?
    a. $\overline{CA} \cong \overline{DO}, \overline{AT} \cong \overline{OG}, \angle A \cong \angle O$
    b. $\overline{CA} \cong \overline{DO}, \overline{AT} \cong \overline{OG}, \angle T \cong \angle G$
    c. $\overline{CA} \cong \overline{DO}, \overline{AT} \cong \overline{OG}, \overline{TC} \cong \overline{GD}$
    d. $\overline{CA} \cong \overline{DO}, \angle T \cong \angle G, \angle A \cong \angle O$
    e. All of the above would show that $\Delta CAT \cong \Delta DOG$

19. To construct a circle given any three non-collinear points in a plane, which of these facts do you need to use?
    a. That three collinear points would define a line.
    b. That if the hypotenuse of a right triangle is the diameter of a circle, the third vertex also lies on the circle.
    c. That the radius of a circle is perpendicular to a line tangent to the circle at the same point.
    d. That the perpendicular bisectors of any two chords always intersect in the center of the circle.
    e. None of the above

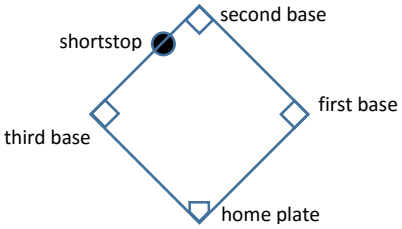
20. The residents of Funny River, Alaska, are considering building a bridge over the town's namesake river. Determine the distance across the river.



    a. 46 ft
    b. 60 ft
    c. 77 ft
    d. 91 ft
    e. None of the above

21. Which of the following shapes **does not** have the same area as the rest?
    a. A 3,4,5, right triangle
    b. A rectangle with a length of 2 and a width of 6
    c. A parallelogram with a base of 1 and a height of 12
    d. A trapezoid with an upper base of 2, a lower base of 6, and a height of 3
    e. They all have the same area

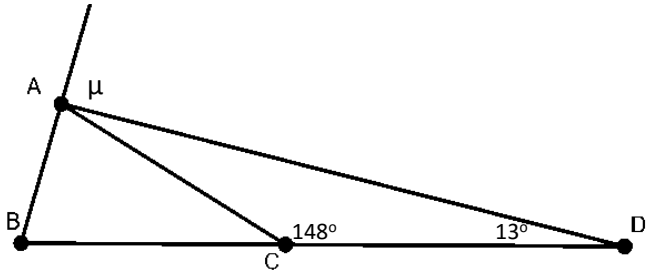
22. In baseball, the bases and home plate form a square that is 90 feet on each side. If the shortstop fields a ground ball exactly one third of the way from second base to third base and throws the ball to first base, to the nearest foot, how far must his throw travel?



a. 101 ft
b. 108 ft
c. 127 ft
d. 95 ft
e. None of the above

23. At Pizza Planet, an 8-inch diameter pizza is cut into 8 slices, and each slice is estimated to contain 80 calories. If a 12-inch diameter pizza is cut into 8 slices, approximately how many calories would each slice contain?
a. 120 calories
b. 220 calories
c. 180 calories
d. 140 calories
e. None of the above

24. In the picture below, if $\overline{AC} \cong \overline{BC}$, find the measure of angle μ.



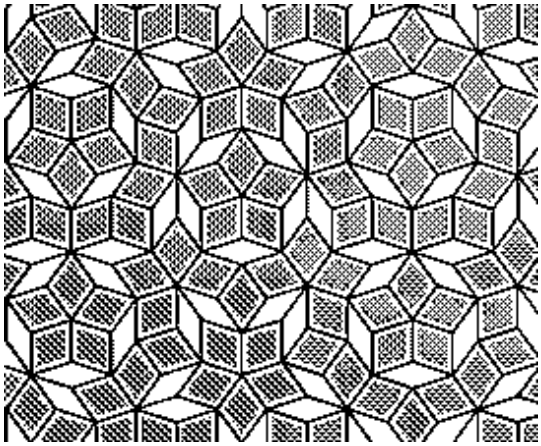
a. 87°
b. 74°
c. 80°
d. 91°
e. None of the above

25. A rectangle has a side of length 6 and a diagonal of length 10. A rhombus also has a side of length 6 and a diagonal of length 10. What is the difference between their areas, to the nearest tenth?
a. 13.2
b. 12.0
c. 14.8
d. Cannot be determined
e. None of the above

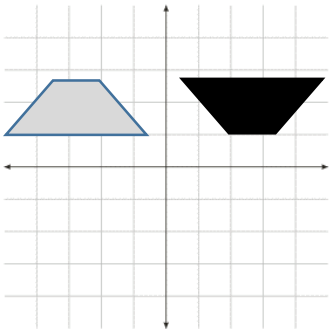
26. The interior angle sum of a _____ is 1800°.
    a. Heptagon
    b. Octagon
    c. Nonagon
    d. Decagon
    e. None of the above

27. A Penrose tiling is a design created by Sir Roger Penrose in which two rhombi (or rhombuses if you prefer, both are correct) tile the plane in a non-periodic fashion (ie. the pattern created cannot repeat itself). Below is an example of a portion of one such tiling. In it, all of the grey rhombi are congruent to each other, and all of the white rhombi are congruent to each other. What is the largest angle in each white rhombus?

    a. 108°
    b. 144°
    c. 120°
    d. 150°
    e. None of the above



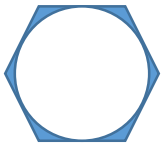
28. Which single transformation of the plane would take the grey trapezoid to the black trapezoid?



    a. Reflection
    b. Translation
    c. Rotation
    d. Reflection OR Rotation
    e. Rotation OR Translation

29. The diagonals of a given parallelogram are 12 cm and 16 cm. What is the longest length possible for one of the parallelogram's sides?
    a. Any value up to 4 cm
    b. Any value up to 20 cm
    c. Any value up to 28 cm
    d. Any value up to 14 cm
    e. None of the above

30. To save on manufacturing costs, Lead It Go, the leading pencil manufacturer in Poughkeepsie, is planning on slimming down their hexagonal pencil design into a circular one, as shown in the picture below. If their current regular hexagonal pencils are 4mm on each side, what will the radius of the circular pencils be?

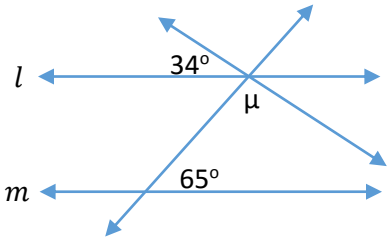


a. $4\sqrt{2}$ mm
b. $4\sqrt{3}$ mm
c. $2\sqrt{2}$ mm
d. $2\sqrt{3}$ mm
e. None of the above

31. The front, side, and top view of a three dimensional shape are given below. What shape is it?

a. Cylinder
b. Prism
c. Cone
d. Pyramid
e. Sphere

32. Given that $l \parallel m$, what is the measure of angle μ?



a. 81°
b. 149°
c. 99°
d. 31°
e. None of the above.