Indiana State Math Contest

2019

Algebra II

This test was prepared by faculty at the University of Southern Indiana

Do not open this test booklet until you have been advised to do so by the test proctor.

1. Solve the inequality for x: $\dfrac{x+7}{x-1} \geq 2x+5$

   a. $(-\infty,-3]\cup[2,\infty)$ b. $(-\infty,1)\cup(1,2]$ c. $(-6,\infty)$ d. $(-\infty,-3]\cup(1,2]$ e. None of these

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2. Find the remainder when $2x^3-4x+6$ is divided by $x-3$

   a. -36 b. -2 c. 12 d. 36 e. 48

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3. Two lines, both passing through $(1,2)$, have slopes whose sum is 3. If one line passes through $(4,y_1)$ and the other passes through $(4,y_2)$, find the sum y_1+y_2.

   a. 5 b. 6 c. 10 d. 13 e. 15

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4. Simplify $\dfrac{x^{-1}y^{-1}}{x^{-1}+y^{-1}}$

   a. $\dfrac{1}{x+y}$ b. 1 c. xy d. $x+y$ e. $\dfrac{x+y}{xy}$

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5. How many solutions does $\Big|\big|5x+4\big|+3x+2\Big|+1=0$ have?

   a. 0 b. 1 c. 2 d. 3 e. 4

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6. If $x^2+xy+y^2=a$ and $x+y=b$, find the value of xy in terms of a and b .

   a. a^2+b b. a^2-b c. a^2+b^2 d. b^2-a^2 e. b^2-a

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7. A quadratic function with vertex $(4,-5)$ contains the point $(-3,1)$. Which of these other points does it contain?

   a. $(18,7)$ b. $(18,13)$ c. $(18,19)$ d. $(18,25)$ e. None of these

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8. A class of 42 students average 76 on a Final Exam. If the scores of the freshmen, who averaged 72, are ignored then the class average is 79. How many freshmen took the Exam?

   a. 3 b. 6 c. 14 d. 18 e. None of these

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9. If $f(x)$ is a linear function with slope of $\frac{3}{2}$, which of these is true of $f^{-1}(x)$?

   a. $m=-\dfrac{2}{3}$ b. $m=\dfrac{2}{3}$ c. $m=-\dfrac{3}{2}$ d. $m=\dfrac{3}{2}$ e. None of these

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10. Simplify $\sqrt[4]{x^{15}} \cdot \sqrt[5]{x^{12}}$

a. x^3 b. x^6 c. x^9 d. x^{12} e. None of these

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11. What is $(1+i)^6$ when written in $a+bi$ form ?

a. $1+i$ b. $6+6i$ c. $6-6i$ d. $0-8i$ e. None of these

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12. What is the product of the solutions to the equation $(x^2-8)^2 - 5(x^2-8) + 4 = 0$?

a. -9 b. -8 c. 0 d. 108 e. None of these

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13. If a graph containing the point $(0,5)$ is reflected across the line $y = 2x$, what point must be on the reflection?

a. $(0,-5)$ b. $(5,0)$ c. $\left(\frac{5}{2},0\right)$ d. $(0,1)$ e. $(4,3)$

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14. At a market, 15 apples are worth 14 bananas, 9 bananas are worth 4 cantaloupes, and 7 cantaloupes are worth 3 dragonfruit. How many apples are 8 dragonfruit worth?

a. 5 b. 30 c. 45 d. 60 e. 120

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15. If $x = -y$, how many of the following statements are **always** true?

$$x^2 = (-y)^2 \qquad x^3 = (-y)^3 \qquad y = -\sqrt{x^2} \qquad |x| = |y|$$

a. 0 b. 1 c. 2 d. 3 e. 4

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16. How many integers are in the solution set of $x^2 + 19x \leq 150$?

a. 19 b. 31 c. 150 d. An infinite number e. None of these

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17. What is the domain of $f(x) = \sqrt{\dfrac{x+2}{x^2-16}}$?

a. $(-4,-2] \cup (4,\infty)$ b. $[-2,\infty)$ c. $[-2,4) \cup (4,\infty)$ d. $(-4,\infty)$ e. None of these

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18. A frog is at one end of a pond and wants to hop to the other. On each jump, she covers $\frac{1}{3}$ of the distance from her current position to the other side. How many jumps will she need to get at least 99% of the way across?

a. 3 b. 12 c. 33 d. 99 e. She will never reach 99%

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19. Which interest rate and compounding period give the best return?

    a. 6% compounded annually
    b. 5.9% compounded quarterly
    c. 5.8% compounded monthly
    d. 5.7% compounded daily
    e. 5.6% compounded continuously

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20. Which of the following values of k will make $\begin{Bmatrix} x^2+y^2=2 \\ y=kx+4 \end{Bmatrix}$ have exactly one solution for x ?

    a. -3 b. 0 c. 9 d. $\sqrt{7}$ e. None of these

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21. You select three marbles, without replacement, from a bag containing 2 white, 3 red, and 5 blue marbles. What is the probability that you select one of each color?

    a. $\frac{1}{30}$ b. $\frac{1}{10}$ c. $\frac{1}{4}$ d. $\frac{3}{10}$ e. $\frac{1}{3}$

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22. Solve $\log_2\left(x^2-6x\right)-\log_2\left(1-x\right)=3$

    a. $-4, 2$ b. 2 c. -4 d. 4 e. None of these

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23. Astronaut Ann looked out her space shuttle window and saw a group of Martians (each with 3 arms and 4 legs) and Venusians (each with 5 arms and 3 legs) playing volleyball on an asteroid. She counted a total of 71 arms and 58 legs. How many aliens were playing volleyball?

    a. 13 b. 15 c. 17 d. 19 e. 20

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24. The graph of $f(x)$ includes the point $(5,-1)$. Which of these numbers **must** be a coordinate of a point on the graph of $f(x-8)$?

    a. -9 b. -3 c. 7 d. 13 e. None of these

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25. Find the reciprocal of $2-3i$.

    a. $\frac{2+3i}{13}$ b. $\frac{-2-3i}{5}$ c. $2+3i$ d. $\frac{2-3i}{13}$ e. $\frac{2+3i}{5}$

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26. How many zeros are at the end of $2019!$ when it is written as an integer?

    a. 201 b. 403 c. 502 d. 2019 e. None of these

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27. Two bowls contain only red and white marbles. The second bowl has twice as many total marbles and twice the probability of drawing a red marble if a single marble is randomly drawn from each bowl. Which of the following statements is true?

    a. The second bowl has half as many red marbles as the first.
    b. The bowls have the same number of red marbles.
    c. The second bowl has twice as many red marbles as the first.
    d. The second bowl has three times as many red marbles as the first.
    e. The second bowl has four times as many red marbles as the first.

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28. If $f(g(x)) = \dfrac{5-x}{2x+3}$ and $g(x) = 2x+1$, which of these could be $f(x)$?

    a. $\dfrac{11-x}{2x+4}$ b. $\dfrac{4-2x}{4x+5}$ c. $\dfrac{5+9x-2x^2}{4x^2+8x+3}$ d. $\dfrac{13}{2x+3}$ e. None of these

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29. The height of a projectile launched with initial upward velocity v_0 and initial height h_0 is given by the function $h(t) = -16t^2 + v_0 t + h_0$. A projectile is launched from a height of 64 feet and lands after 8 seconds. What is the maximum height of the projectile?

    a. 120 b. 289 c. 360 d. 512 e. None of these

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30. A student added all the page numbers in a book from 1 to the last page and got a total of 2019. They were told this is not possible, so they went back and found that two pages had stuck together causing them to skip a pair of consecutive numbers. Which two numbers were skipped?

    a. 8&9 b. 19&20 c. 30&31 d. 41&42 e. None of these

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31. If you flip an unfair coin twice, the chance that you get heads once and tails once is 42%. How is the coin weighted?

    a. 42:58 b. 21:72 c. 30:70 d. 40:60 e. None of these

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32. For what value of C is $f(x) = \begin{cases} x^2 + Cx, & x < 2 \\ 7 - 3x, & x \geq 2 \end{cases}$ a continuous function?

    a. -2 b. $-\dfrac{3}{2}$ c. 0 d. 2 e. None of these
