



# **Indiana State Math Contest 2017**

## **Geometry**

This test was prepared by faculty at  
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Do not open this test booklet until you have  
been advised to do so by the test proctor.

1. The angles of a certain quadrilateral are  $x$ ,  $x + 10^\circ$ ,  $x + 20^\circ$ , and  $x + 30^\circ$ . Which of the following is equal to  $x$ ?
  - (a)  $70^\circ$
  - (b)  $80^\circ$
  - (c)  $30^\circ$
  - (d)  $45^\circ$
  - (e) None of the above.
  
2. The length of a certain rectangle is twice its width. The diagonal of the rectangle has length  $d$ . Find the area of the rectangle in terms of  $d$ .
  - (a)  $\frac{2d^2}{3}$
  - (b)  $\frac{d^2}{6}$
  - (c)  $\frac{2d^2}{5}$
  - (d)  $\frac{3d^2}{4}$
  - (e) None of the above.
  
3. The three points  $(1, 2)$ ,  $(3, 7)$ , and  $(k, 10)$  are collinear. Which of the following is equal to  $k$ ?
  - (a)  $\frac{11}{3}$
  - (b)  $\frac{41}{9}$
  - (c)  $\frac{21}{5}$
  - (d)  $\frac{31}{7}$
  - (e) None of the above.
  
4. The circumference of a circle is 3 in. Find the exact area of the circle.
  - (a)  $\frac{9}{\pi} \text{ in}^2$
  - (b)  $\frac{36}{\pi} \text{ in}^2$
  - (c)  $\frac{9}{16\pi} \text{ in}^2$
  - (d)  $\frac{9}{4\pi} \text{ in}^2$
  - (e) None of the above.

5. The area of a certain square is numerically the same as its perimeter. What is the area of this square?
- (a) 4
  - (b) 2
  - (c) 1
  - (d) 16
  - (e) None of the above.
6. A rectangular box has a length of  $a$ , width of  $b$ , and a height of  $c$ . Which of the following represents the length of one of its interior diagonals?
- (a)  $\frac{\sqrt{a^2+b^2}+\sqrt{a^2+c^2}+\sqrt{b^2+c^2}}{3}$
  - (b)  $\frac{a+b+c}{3}$
  - (c)  $\sqrt{a^2 + b^2 + c^2}$
  - (d)  $\sqrt{a^2 + b^2} + \sqrt{a^2 + c^2} + \sqrt{b^2 + c^2}$
  - (e) None of the above.
7. An angle measures 2 radians. What is its measure in degrees?
- (a)  $\left(\frac{360}{\pi}\right)^\circ$
  - (b)  $\left(\frac{720}{\pi}\right)^\circ$
  - (c)  $\left(\frac{\pi}{360}\right)^\circ$
  - (d)  $\left(\frac{\pi}{90}\right)^\circ$
  - (e) None of the above.
8. Which of the following statements about a triangle is FALSE?
- (a) The three medians are concurrent.
  - (b) The three altitudes are concurrent.
  - (c) The three angle bisectors are concurrent.
  - (d) The perpendicular bisectors of each side are concurrent.
  - (e) None of the above.

9. Which of the following represents a line through the origin parallel to the line  $2x - y = 1$ ?
- (a)  $x + 2y = 0$
  - (b)  $x - 2y = 0$
  - (c)  $2x + y = 0$
  - (d)  $2x - y = 0$
  - (e) None of the above.
10. Two sides of a triangle measure 3 in and 5 in. The angle between these two sides measures  $120^\circ$ . Find the exact length of the side opposite the  $120^\circ$  angle.
- (a) 7 in
  - (b)  $4 + 2\sqrt{3}$  in
  - (c)  $4\sqrt{3}$  in
  - (d)  $\sqrt{19}$  in
  - (e) None of the above.
11. The lengths of the sides of a right triangle are all integers. Suppose one leg has length 12. Which of the following **cannot** be the length of the hypotenuse?
- (a) 15
  - (b) 17
  - (c) 20
  - (d) 37
  - (e) 13
12. The sum of the lengths of the edges of a cube is  $L$ . Find the volume of this cube.
- (a)  $\frac{L^3}{4096}$
  - (b)  $\frac{L^3}{1728}$
  - (c)  $\frac{L^3}{512}$
  - (d)  $\frac{L^3}{27}$
  - (e) None of the above.

13. A convex polyhedron has 5 vertices and 6 faces. How many edges does it have?
- (a) 7
  - (b) 9
  - (c) 8
  - (d) 10
  - (e) None of the above.
14. The set of all points in three dimensions that are equidistant from a single point
- (a) is a sphere.
  - (b) is a plane.
  - (c) lie on two parallel planes.
  - (d) is a line.
  - (e) None of the above.
15. Which of the following statements is FALSE:
- (a) A square is a rhombus.
  - (b) Every parallelogram is a rhombus.
  - (c) The angles in a rhombus sum to  $360^\circ$ .
  - (d) Opposite sides of a rhombus are congruent.
  - (e) None of the above.
16. The diameter of a circle is  $d$ . Another circle has twice the area of this circle. What is its diameter?
- (a)  $4d$
  - (b)  $\frac{d\sqrt{2}}{2}$
  - (c)  $2d$
  - (d)  $d\sqrt{2}$
  - (e) None of the above.

17. The base angles of a trapezoid are each  $60^\circ$ . The bottom base has length 7 in and the top base has length 3 in. Find the area of the trapezoid.
- (a)  $5 \text{ in}^2$
  - (b)  $10\sqrt{3} \text{ in}^2$
  - (c)  $5\sqrt{3} \text{ in}^2$
  - (d)  $10 \text{ in}^2$
  - (e) None of the above.
18. Two angles of an isosceles triangle are  $x$  and  $x + 18^\circ$ . Two of the angles of this triangle are greater than  $60^\circ$ . Find  $x$ .
- (a)  $72^\circ$
  - (b)  $48^\circ$
  - (c)  $66^\circ$
  - (d)  $54^\circ$
  - (e) None of the above.
19. How many  $3 \text{ in} \times 3 \text{ in}$  square tiles are needed to cover a square that is  $2 \text{ ft} \times 2 \text{ ft}$ ?
- (a) 64
  - (b) 16
  - (c) 32
  - (d) 144
  - (e) None of the above.
20. The legs of a right triangle have lengths 15 in and 20 in. Find the length of the altitude from the vertex of the right angle to the hypotenuse.
- (a) 12 in
  - (b) 24 in
  - (c) 6 in
  - (d) 8 in
  - (e) None of the above.

21. Each individual square on a sheet of graph paper is 1 unit  $\times$  1 unit. A square is drawn on this sheet of graph paper using grid points as its corner points. Which of the following **cannot** be the area of this square?
- (a) 10 units<sup>2</sup>
  - (b) 13 units<sup>2</sup>
  - (c) 7 units<sup>2</sup>
  - (d) 9 units<sup>2</sup>
  - (e) 8 units<sup>2</sup>
22. A cylindrical tank of radius 6 in and height 24 in is full of water. All of the water from the tank is poured into an empty glass aquarium that has the shape of a rectangular prism with a base measuring 12 in  $\times$  24 in and a height of 54 in. After the water is poured in the aquarium, what is the height of the water in the aquarium?
- (a)  $\frac{3\pi}{2}$  in
  - (b)  $12\pi$  in
  - (c)  $6\pi$  in
  - (d)  $\frac{\pi}{3}$  in
  - (e) None of the above.
23. The length of a rectangle is 1 in less than three times its width. Its perimeter is 10 in. Find the area of this rectangle.
- (a) 20 in<sup>2</sup>
  - (b)  $\frac{46}{3}$  in<sup>2</sup>
  - (c)  $\frac{21}{4}$  in<sup>2</sup>
  - (d)  $\frac{319}{16}$  in<sup>2</sup>
  - (e) None of the above.
24. The lengths of the sides of a certain triangle are 12, 17, and  $x$  where  $x$  is an integer. How many different values for  $x$  are possible?
- (a) 25
  - (b) 21
  - (c) 22
  - (d) 23
  - (e) None of the above.

25. Find the coordinates of the point obtained by reflecting the point  $(-2, 5)$  about the  $y$ -axis.
- (a)  $(-2, -5)$
  - (b)  $(-5, -2)$
  - (c)  $(2, 5)$
  - (d)  $(5, -2)$
  - (e) None of the above.
26. The base angles of an isosceles triangle are each  $30^\circ$  and the length of its longest side is 10 in. Find the area of this triangle.
- (a)  $25\sqrt{3} \text{ in}^2$
  - (b)  $\frac{25\sqrt{3}}{2} \text{ in}^2$
  - (c)  $\frac{25\sqrt{3}}{3} \text{ in}^2$
  - (d)  $\frac{50\sqrt{3}}{3} \text{ in}^2$
  - (e) None of the above.
27. An equilateral triangle has an area of  $\sqrt{3} \text{ in}^2$ . Find its perimeter.
- (a)  $\frac{9}{2} \text{ in}$
  - (b)  $\frac{3}{2} \text{ in}$
  - (c) 4 in
  - (d) 3 in
  - (e) None of the above.
28. A right triangle has legs of lengths 2 in and  $\frac{5}{6}$  in. Find the length of the hypotenuse.
- (a)  $\frac{13}{6} \text{ in}$
  - (b)  $\frac{17}{6} \text{ in}$
  - (c)  $\frac{\sqrt{119}}{6} \text{ in}$
  - (d)  $\frac{7\sqrt{6}}{6} \text{ in}$
  - (e) None of the above.



29. The surface area of a cube is  $100 \text{ in}^2$ . Suppose each edge of the cube is doubled in length. Find the surface area of the resulting cube.
- (a)  $200\sqrt{2} \text{ in}^2$
  - (b)  $200 \text{ in}^2$
  - (c)  $100\sqrt{2} \text{ in}^2$
  - (d)  $400 \text{ in}^2$
  - (e) None of the above.
30. Which of the following statements is FALSE:
- (a) Complements of the same angle are congruent.
  - (b) A supplement of an acute angle must be an obtuse angle.
  - (c) Vertical angles sum to  $90^\circ$ .
  - (d) Supplements of the same angle are congruent.
  - (e) None of the above.
31. Which of the following statements is FALSE:
- (a) The triangle with sides 2, 3, and 4 is similar to the triangle with sides 4, 5, and 6.
  - (b) Any two equilateral triangles are similar.
  - (c) Any two isosceles right triangles are similar.
  - (d) The right triangle with sides 3, 4, and 5 is similar to the right triangle with sides 9, 12, and 15.
  - (e) None of the above.
32. The diagonals of a rhombus are 6 in and 8 in. What is the length of each side of the rhombus?
- (a)  $4\sqrt{2} \text{ in}$
  - (b) 5 in
  - (c)  $6\sqrt{2} \text{ in}$
  - (d) 7 in
  - (e) None of the above.

33. A sector with central angle  $90^\circ$  is cut from a circle of radius 4 in. The edges of the sector are taped together to form a right circular cone. Find the volume of this cone.

(a)  $\frac{8\pi\sqrt{3}}{3}$  in<sup>3</sup>

(b)  $\frac{\pi\sqrt{3}}{3}$  in<sup>3</sup>

(c)  $\frac{\pi\sqrt{15}}{3}$  in<sup>3</sup>

(d)  $\frac{4\pi\sqrt{15}}{3}$  in<sup>3</sup>

(e) None of the above.

34. A triangle has vertices  $(0, 0)$ ,  $(4, 0)$ , and  $(0, 4)$ . A vertical line through  $(b, 0)$  divides the triangle into two regions of equal area. Find  $b$ .

(a)  $4 - 2\sqrt{2}$

(b)  $\sqrt{2}$

(c)  $4 - \sqrt{2}$

(d)  $2\sqrt{2}$

(e) None of the above.

35. A triangle with sides 6 in, 8 in, and 10 in is inscribed in a circle. Find the radius of this circle.

(a)  $4\sqrt{2}$  in

(b) 5 in

(c)  $6\sqrt{3}$  in

(d) 12 in

(e) None of the above.