



Indiana State Math Contest 2017

Geometry

This test was prepared by faculty at
Indiana University - Purdue University
Columbus

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been advised to do so by the test proctor.

1. The angles of a certain quadrilateral are x , $x + 10^\circ$, $x + 20^\circ$, and $x + 30^\circ$. Which of the following is equal to x ?
 - (a) 70°
 - (b) 80°
 - (c) 30°
 - (d) 45°
 - (e) None of the above.

2. The length of a certain rectangle is twice its width. The diagonal of the rectangle has length d . Find the area of the rectangle in terms of d .
 - (a) $\frac{2d^2}{3}$
 - (b) $\frac{d^2}{6}$
 - (c) $\frac{2d^2}{5}$
 - (d) $\frac{3d^2}{4}$
 - (e) None of the above.

3. The three points $(1, 2)$, $(3, 7)$, and $(k, 10)$ are collinear. Which of the following is equal to k ?
 - (a) $\frac{11}{3}$
 - (b) $\frac{41}{9}$
 - (c) $\frac{21}{5}$
 - (d) $\frac{31}{7}$
 - (e) None of the above.

4. The circumference of a circle is 3 in. Find the exact area of the circle.
 - (a) $\frac{9}{\pi} \text{ in}^2$
 - (b) $\frac{36}{\pi} \text{ in}^2$
 - (c) $\frac{9}{16\pi} \text{ in}^2$
 - (d) $\frac{9}{4\pi} \text{ in}^2$
 - (e) None of the above.

5. The area of a certain square is numerically the same as its perimeter. What is the area of this square?
- (a) 4
 - (b) 2
 - (c) 1
 - (d) 16
 - (e) None of the above.
6. A rectangular box has a length of a , width of b , and a height of c . Which of the following represents the length of one of its interior diagonals?
- (a) $\frac{\sqrt{a^2+b^2}+\sqrt{a^2+c^2}+\sqrt{b^2+c^2}}{3}$
 - (b) $\frac{a+b+c}{3}$
 - (c) $\sqrt{a^2 + b^2 + c^2}$
 - (d) $\sqrt{a^2 + b^2} + \sqrt{a^2 + c^2} + \sqrt{b^2 + c^2}$
 - (e) None of the above.
7. An angle measures 2 radians. What is its measure in degrees?
- (a) $\left(\frac{360}{\pi}\right)^\circ$
 - (b) $\left(\frac{720}{\pi}\right)^\circ$
 - (c) $\left(\frac{\pi}{360}\right)^\circ$
 - (d) $\left(\frac{\pi}{90}\right)^\circ$
 - (e) None of the above.
8. Which of the following statements about a triangle is FALSE?
- (a) The three medians are concurrent.
 - (b) The three altitudes are concurrent.
 - (c) The three angle bisectors are concurrent.
 - (d) The perpendicular bisectors of each side are concurrent.
 - (e) None of the above.

9. Which of the following represents a line through the origin parallel to the line $2x - y = 1$?
- (a) $x + 2y = 0$
 - (b) $x - 2y = 0$
 - (c) $2x + y = 0$
 - (d) $2x - y = 0$
 - (e) None of the above.
10. Two sides of a triangle measure 3 in and 5 in. The angle between these two sides measures 120° . Find the exact length of the side opposite the 120° angle.
- (a) 7 in
 - (b) $4 + 2\sqrt{3}$ in
 - (c) $4\sqrt{3}$ in
 - (d) $\sqrt{19}$ in
 - (e) None of the above.
11. The lengths of the sides of a right triangle are all integers. Suppose one leg has length 12. Which of the following **cannot** be the length of the hypotenuse?
- (a) 15
 - (b) 17
 - (c) 20
 - (d) 37
 - (e) 13
12. The sum of the lengths of the edges of a cube is L . Find the volume of this cube.
- (a) $\frac{L^3}{4096}$
 - (b) $\frac{L^3}{1728}$
 - (c) $\frac{L^3}{512}$
 - (d) $\frac{L^3}{27}$
 - (e) None of the above.

13. A convex polyhedron has 5 vertices and 6 faces. How many edges does it have?
- (a) 7
 - (b) 9
 - (c) 8
 - (d) 10
 - (e) None of the above.
14. The set of all points in three dimensions that are equidistant from a single point
- (a) is a sphere.
 - (b) is a plane.
 - (c) lie on two parallel planes.
 - (d) is a line.
 - (e) None of the above.
15. Which of the following statements is FALSE:
- (a) A square is a rhombus.
 - (b) Every parallelogram is a rhombus.
 - (c) The angles in a rhombus sum to 360° .
 - (d) Opposite sides of a rhombus are congruent.
 - (e) None of the above.
16. The diameter of a circle is d . Another circle has twice the area of this circle. What is its diameter?
- (a) $4d$
 - (b) $\frac{d\sqrt{2}}{2}$
 - (c) $2d$
 - (d) $d\sqrt{2}$
 - (e) None of the above.

17. The base angles of a trapezoid are each 60° . The bottom base has length 7 in and the top base has length 3 in. Find the area of the trapezoid.
- (a) 5 in^2
 - (b) $10\sqrt{3} \text{ in}^2$
 - (c) $5\sqrt{3} \text{ in}^2$
 - (d) 10 in^2
 - (e) None of the above.
18. Two angles of an isosceles triangle are x and $x + 18^\circ$. Two of the angles of this triangle are greater than 60° . Find x .
- (a) 72°
 - (b) 48°
 - (c) 66°
 - (d) 54°
 - (e) None of the above.
19. How many $3 \text{ in} \times 3 \text{ in}$ square tiles are needed to cover a square that is $2 \text{ ft} \times 2 \text{ ft}$?
- (a) 64
 - (b) 16
 - (c) 32
 - (d) 144
 - (e) None of the above.
20. The legs of a right triangle have lengths 15 in and 20 in. Find the length of the altitude from the vertex of the right angle to the hypotenuse.
- (a) 12 in
 - (b) 24 in
 - (c) 6 in
 - (d) 8 in
 - (e) None of the above.

21. Each individual square on a sheet of graph paper is 1 unit \times 1 unit. A square is drawn on this sheet of graph paper using grid points as its corner points. Which of the following **cannot** be the area of this square?
- (a) 10 units²
 - (b) 13 units²
 - (c) 7 units²
 - (d) 9 units²
 - (e) 8 units²
22. A cylindrical tank of radius 6 in and height 24 in is full of water. All of the water from the tank is poured into an empty glass aquarium that has the shape of a rectangular prism with a base measuring 12 in \times 24 in and a height of 54 in. After the water is poured in the aquarium, what is the height of the water in the aquarium?
- (a) $\frac{3\pi}{2}$ in
 - (b) 12π in
 - (c) 6π in
 - (d) $\frac{\pi}{3}$ in
 - (e) None of the above.
23. The length of a rectangle is 1 in less than three times its width. Its perimeter is 10 in. Find the area of this rectangle.
- (a) 20 in²
 - (b) $\frac{46}{3}$ in²
 - (c) $\frac{21}{4}$ in²
 - (d) $\frac{319}{16}$ in²
 - (e) None of the above.
24. The lengths of the sides of a certain triangle are 12, 17, and x where x is an integer. How many different values for x are possible?
- (a) 25
 - (b) 21
 - (c) 22
 - (d) 23
 - (e) None of the above.

25. Find the coordinates of the point obtained by reflecting the point $(-2, 5)$ about the y -axis.
- (a) $(-2, -5)$
 - (b) $(-5, -2)$
 - (c) $(2, 5)$
 - (d) $(5, -2)$
 - (e) None of the above.
26. The base angles of an isosceles triangle are each 30° and the length of its longest side is 10 in. Find the area of this triangle.
- (a) $25\sqrt{3} \text{ in}^2$
 - (b) $\frac{25\sqrt{3}}{2} \text{ in}^2$
 - (c) $\frac{25\sqrt{3}}{3} \text{ in}^2$
 - (d) $\frac{50\sqrt{3}}{3} \text{ in}^2$
 - (e) None of the above.
27. An equilateral triangle has an area of $\sqrt{3} \text{ in}^2$. Find its perimeter.
- (a) $\frac{9}{2} \text{ in}$
 - (b) $\frac{3}{2} \text{ in}$
 - (c) 4 in
 - (d) 3 in
 - (e) None of the above.
28. A right triangle has legs of lengths 2 in and $\frac{5}{6}$ in. Find the length of the hypotenuse.
- (a) $\frac{13}{6} \text{ in}$
 - (b) $\frac{17}{6} \text{ in}$
 - (c) $\frac{\sqrt{119}}{6} \text{ in}$
 - (d) $\frac{7\sqrt{6}}{6} \text{ in}$
 - (e) None of the above.

29. The surface area of a cube is 100 in^2 . Suppose each edge of the cube is doubled in length. Find the surface area of the resulting cube.
- (a) $200\sqrt{2} \text{ in}^2$
 - (b) 200 in^2
 - (c) $100\sqrt{2} \text{ in}^2$
 - (d) 400 in^2
 - (e) None of the above.
30. Which of the following statements is FALSE:
- (a) Complements of the same angle are congruent.
 - (b) A supplement of an acute angle must be an obtuse angle.
 - (c) Vertical angles sum to 90° .
 - (d) Supplements of the same angle are congruent.
 - (e) None of the above.
31. Which of the following statements is FALSE:
- (a) The triangle with sides 2, 3, and 4 is similar to the triangle with sides 4, 5, and 6.
 - (b) Any two equilateral triangles are similar.
 - (c) Any two isosceles right triangles are similar.
 - (d) The right triangle with sides 3, 4, and 5 is similar to the right triangle with sides 9, 12, and 15.
 - (e) None of the above.
32. The diagonals of a rhombus are 6 in and 8 in. What is the length of each side of the rhombus?
- (a) $4\sqrt{2} \text{ in}$
 - (b) 5 in
 - (c) $6\sqrt{2} \text{ in}$
 - (d) 7 in
 - (e) None of the above.

33. A sector with central angle 90° is cut from a circle of radius 4 in. The edges of the sector are taped together to form a right circular cone. Find the volume of this cone.

(a) $\frac{8\pi\sqrt{3}}{3}$ in³

(b) $\frac{\pi\sqrt{3}}{3}$ in³

(c) $\frac{\pi\sqrt{15}}{3}$ in³

(d) $\frac{4\pi\sqrt{15}}{3}$ in³

(e) None of the above.

34. A triangle has vertices $(0, 0)$, $(4, 0)$, and $(0, 4)$. A vertical line through $(b, 0)$ divides the triangle into two regions of equal area. Find b .

(a) $4 - 2\sqrt{2}$

(b) $\sqrt{2}$

(c) $4 - \sqrt{2}$

(d) $2\sqrt{2}$

(e) None of the above.

35. A triangle with sides 6 in, 8 in, and 10 in is inscribed in a circle. Find the radius of this circle.

(a) $4\sqrt{2}$ in

(b) 5 in

(c) $6\sqrt{3}$ in

(d) 12 in

(e) None of the above.