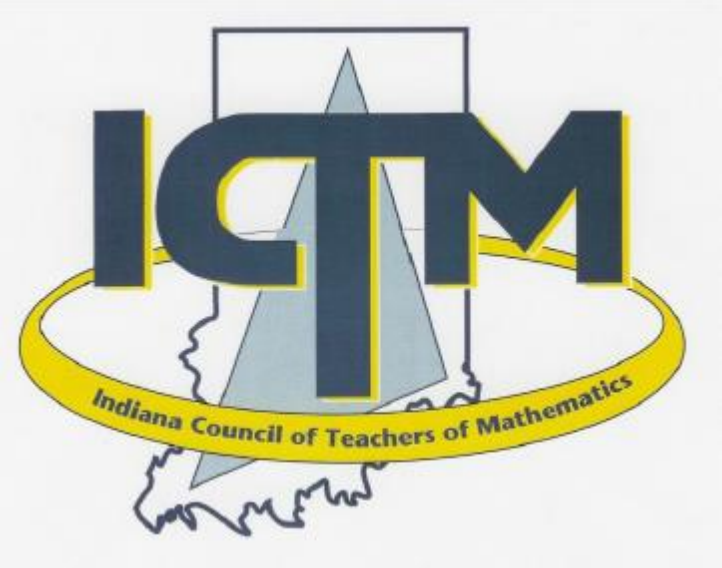


Comprehensive 2017

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Ball State University**

ICTM Website

http://www.indianamath.org/

Do not open this test booklet until you have been advised by the test proctor.

Next year's math contest date:

1. A and B go fishing along the bank of a river. A catches 3 fish and B catches 2 fish. When they are ready to eat the fish C comes and asks to share the fish with them, and so they eat the 5 fish together. Each person eats the same amount of fish. When they finish eating, C thanks them and gives them \$10. How should A and B split this money in the fairest possible way?

   (A) A gets \$6 and B gets \$4
   (B) A gets \$6.50 and B gets \$3.50
   (C) A gets \$7 and B gets \$3
   (D) A gets \$7.50 and B gets \$2.50
   (E) A gets \$8 and B gets \$2

2. A math exam has exactly 3 questions: Q1, Q2, Q3

   You know that:

   i. exactly 25 students took this exam
   ii. each student solved at least 1 question
   iii. each of the 3 questions was solved by at least 1 student
   iv. among the students who could not solve Q1, twice as many solved Q2 as solved Q3
   v. the number of students who could only solve Q1 is exactly 1 more than the number of students who solved Q1 among the rest of the students
   vi. among the students who could only solve 1 question, exactly half of them could not solve Q1

   How many students solved Q1?

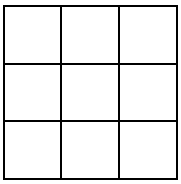
   (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

3. When $1^2+2^2+3^2+\cdots+2016^2+2017^2$ is divided by 3 the remainder is *m*.
   When $1^2+2^2+3^2+\cdots+2016^2+2017^2$ is divided by 5 the remainder is *n*.

   What is the value of $m+n$?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. 9 different positive integers are put into this 3×3 table, exactly 1 integer for each square.



   If the sum of the integers in each of the four 2×2 sub-tables is 100, then what is the minimum value of the sum of all of the 9 different positive integers?

   (A) 118 (B) 121 (C) 125 (D) 128 (E) 131

5. Let M be the point $(-2,-1)$, N be the point $(1,-5)$, and C be the circle $x^2+y^2-4x-2y+1=0$. Let P be a point that is allowed to move on the circle C.

   What is the maximum area of the triangle formed by M, N, and P ?

   (A) 11 (B) 13.5 (C) 16 (D) 18.5 (E) 21

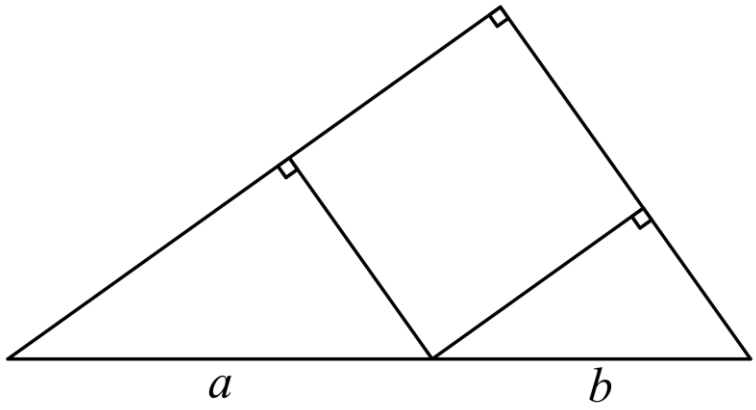
6. If the function f satisfies $f(x+y)=f(x)+f(y)$ for all real numbers *x* and *y*, then f must be:

   (A) an odd function
   (B) an even function
   (C) an increasing function
   (D) a decreasing function
   (E) none of the above

7. The parabola that passes through the points $(-3,-92)$, $(-2,-66)$, and $(-1,-44)$ attains its maximum value when *x* equals:

   (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

8. The large right triangle below is made up of two smaller right triangles and a square. The hypotenuse of the larger right triangle on the left has length *a*, and the hypotenuse of the other small right triangle has length *b*. What is the area of the square?



(A) $\frac{(ab)^2}{a^2+b^2}$ (B) $\frac{(a+b)^2}{(ab)^2}$ (C) $\frac{a^2+b^2}{ab}$ (D) $\frac{(ab)^2}{(a+b)^2}$ (E) $\frac{(a+b)^2}{2ab}$

9. The Federal Information Processing Standards Publication 180-4 specifies that the Secure Hash Algorithm SHA-256 use 64 special constants. This publication states that these 64 special constants are:

"the first 32 bits of the fractional parts of the cube roots of the first 64 prime numbers."

The publication then lists these 64 special constants in order. Each constant is written in HEX, which is a base-16 numerical representation that uses the symbols 0-9 to represent base-10 zero to nine, and A, B, C, D, E, F to represent base-10 ten to fifteen.

For example, the first constant uses "the first 32 bits of the fractional part of the cube root of 2."
This constant is written in HEX as: 428A2F98
As a string of 8 symbols, the first constant contains 6 digits and 2 letters.
The sum of the 6 digits is $4+2+8+2+9+8=33$.

The fifth constant uses "the first 32 bits of the fractional part of the cube root of 11."
Written in HEX, the fifth constant also contains 6 digits and 2 letters.

What is the sum of the 6 digits of the fifth constant when it is written in HEX?

(A) 17 (B) 27 (C) 30 (D) 42 (E) 53

10. A sightseeing boat travels along a river from point A to point B and then back to point A. The distance from A to B is 10 km, and the total travel time from A to B to A is 3 hours. During the entire trip the speed of the boat and the speed of the water remain constant.

If the distance traveled during the first hour is 8 km more than the distance traveled during the third hour, what is the speed of the water?

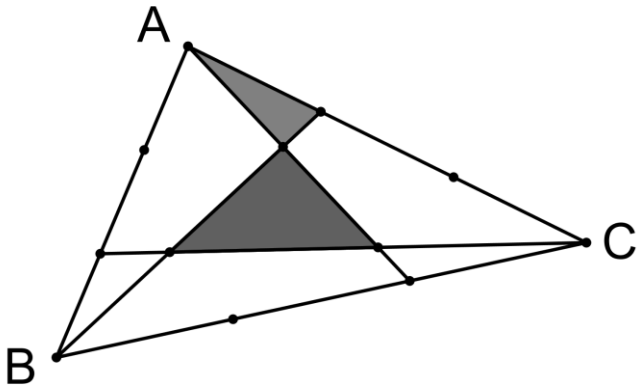
(A) $4\ \frac{\text{km}}{\text{hr}}$ (B) $5\ \frac{\text{km}}{\text{hr}}$ (C) $6\ \frac{\text{km}}{\text{hr}}$ (D) $7\ \frac{\text{km}}{\text{hr}}$ (E) $8\ \frac{\text{km}}{\text{hr}}$

11. There is a large $n\times n\times n$ cube, where n is a positive integer. Each of the 6 faces of this cube are painted either red or black. This cube is then cut into unit cubes. The number of unit cubes with some red paint is exactly 200 more than the number of unit cubes with some black paint.

How many cubes have no paint at all?

(A) 343 (B) 512 (C) 729 (D) 1000 (E) 1331

12. Triangle ABC has area 1. Each of the 3 sides of triangle ABC is divided into thirds and line segments are drawn to produce the picture below. What is the total area of the two shaded regions?



(A) $\frac{4}{21}$ (B) $\frac{5}{21}$ (C) $\frac{2}{7}$ (D) $\frac{1}{3}$ (E) $\frac{8}{21}$

13. Monkey secretly gives positive integer r to Rabbit, positive integer f to Frog, and positive integer t to Turtle. Rabbit, Frog, and Turtle do not know one another's numbers but they know the sum of the three numbers is 14. They then make the following statements:

Rabbit says: I know that Frog and Turtle have different numbers!
then Frog says: Knowing this I now know that all three of our numbers are different!
then Turtle says: Knowing this I now know all three of our numbers!

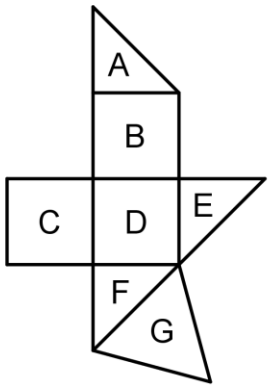
If Rabbit, Frog, and Turtle can reason perfectly, what is the product of their three numbers?

(A) 22 (B) 42 (C) 49 (D) 54 (E) 90

14. How many digits are in the integer 123456^{7890} ?

(A) 4018 (B) 17553 (C) 40173 (D) 92500 (E) 102385

15. In the figure below: A, E, and F are isosceles right triangles
B, C, and D are squares of area 1 each
G is an equilateral triangle



The figure is folded along the edges to form a polyhedron with A, B, C, D, E, F, and G as faces. What is the volume of this polyhedron?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$ (E) $\frac{4}{3}$

16. The value of:

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ \cdots\ \text{and so on}\ \cdots}}}}}}}$$

equals $\frac{a+\sqrt{15}}{b}$, where a and b are relatively prime positive integers.

What is the value of $15a^2+9ab+3b^2$?

(A) 1125 (B) 1431 (C) 1500 (D) 1611 (E) 1695

17. In the following long division problem, most of the digits (26 in fact) are hidden by the symbol X. What is the sum of all of the 26 hidden digits?

```
            X  X  8  X  X
      ------------------
X  X | X  X  X  X  X  X  X
       X  X  X
       ---------
             X  X
             X  X
             ------
                X  X  X
                X  X  X
                ------
                      1
```

(A) 113 (B) 114 (C) 115 (D) 122 (E) 123

18. In the grid below each row, column, and 3×3 box must contain exactly one of each digit from 1 to 9. What is the value of the digit X?

		5					9	
9			6	2		5		
	7			4				3
							3	
	4	7		6		1	2	
	8				X			
8				1			5	
		6		9	7			8
	9					3		

(A) 1 (B) 3 (C) 4 (D) 5 (E) 9

19. How many ordered triples of integers (a, b, c) satisfy both

$$|a+b|+c=19 \text{ and } ab+|c|=97\ ?$$

(A) 0 (B) 6 (C) 12 (D) 20 (E) 24

20. The hour and minute hands of a clock are exactly the same. Both of these hands move continuously and there is no second hand. How many times each day is it impossible to tell what time it is?

(A) 132 (B) 143 (C) 264 (D) 275 (E) 286

21. The school gym has 100 lockers numbered from 1 to 100. The 1st student that arrives opens all of the lockers. The 2nd student that arrives closes every locker that is a multiple of 2. The 3rd student that arrives changes the state of every locker that is a multiple of 3. The 4th student that arrives changes the state of every locker that is a multiple of 4, and so on.

    This continues until a total of 100 students have arrived. What is the sum of the numbers on the lockers that are now open?

    (A) 385 (B) 395 (C) 412 (D) 439 (E) 475

22. A list of integers has mode 32 and mean 22. The smallest number in the list is 10. The median m of the list is a member of the list. If m were replaced by $m+10$, the mean and the median of the new list would be 24 and $m+10$, respectively. If m were instead replaced by $m-8$, the median of the new list would be $m-4$.

    What is the value of m?

    (A) 14 (B) 16 (C) 18 (D) 20 (E) 22

23. A fair 2-headed coin, a fair 2-tailed coin, and a fair ordinary coin are placed in a bag. One of the coins is chosen at random and flipped. The result is "heads."

    What is the probability that the other side of this coin is also a head?

    (A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

24. A refinery blends high and low octane gasoline into three grades: regular, premium, and super-premium. The regular grade consists of 60% high octane and 40% low octane, the premium grade consists of 70% high octane and 30% low octane, and the super-premium grade consists of 80% high octane and 20% low octane.

    The refinery has available 140,000 gallons of high octane and 120,000 gallons of low octane, but can mix only 225,000 gallons. They can sell the regular gasoline to a local gas station for $1.20 per gallon, the premium for $1.30 per gallon, and the super-premium for $1.40 per gallon.

    What is the maximum possible revenue?

    (A) $275,000 (B) $280,000 (C) $285,000 (D) $290,000 (E) $295,000