Pre-Algebra

2012

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

ICTM Website
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Next year's math contest date: April 27, 2013

1. The average of $\frac{1}{20}, \frac{2}{30}$, and $\frac{3}{40}$ is:

A) $\frac{1}{15}$ B) $\frac{23}{360}$ C) $\frac{23}{120}$ D) $\frac{23}{240}$ E) none of these

2. The number halfway between $0.\overline{45}$ and $\frac{7}{12}$ is:

A) $\frac{37}{72}$ B) $\frac{137}{132}$ C) $\frac{1}{2}$ D) $\frac{31}{60}$ E) none of these

3. Mikaila works at a movie theater. If sales tax is 5%, how much should she charge for an enormous popcorn so that the total cost, including sales tax is $10.50?

A) $10   B) $9.95   C) $9.90   D) $9.86   E) none of these

4. A class of 200 students averaged 66% on an examination; another class of 300 students averaged 56%. The average percentage for all students was:

A) 63   B) 62   C) 61   D) 60   E) none of these

5. The difference of the smallest even integer with exactly 6 distinct positive factors and the smallest odd integer with exactly 6 distinct positive factors is:

A) 13   B) 33   C) 211   D) 665   E) none of these

6. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 62 cm, 83 cm, and 95 cm. The area of the square, in cm^2 is:

A) 2400   B) 3600   C) 6800   D) 6400   E) 14400

7. If the pattern INDIANAMATHINDIANAMATHINDIANAMATH… is continued, the 2012th letter in the pattern is:

A) I   B) N   C) D   D) M   E) none of these

8.   A bag contains green, yellow, and orange marbles. The ratio of green to yellow marbles is 2:5. The ratio of yellow to orange marbles is 3:4. The ratio of green marbles to orange marbles is:

      A) $2:1$ B) $1:2$ C) $10:3$ D) $3:10$ E) none of these

9.   If the length and width of a rectangle are each increased by 10%, then the area of the rectangle is increased by:

      A) 1%   B) 10%   C) 20%   D) 21%   E) 40%

10.   Mr. Green receives a 10% raise every year. His salary after four such raises has gone up by what percent?

      A) 40   B) 44   C) 45   D) more than 45   E) none of these

11.   In the product $B2 \times 7B = 6396$, B is a digit. The value of B is equal to:

      A) 8   B) 7   C) 6   D) 5   E) none of these

12.   If $a * b = a^2 - b$, then $(3 * 5) * 8$ is:

      A) 8   B) -8   C) -6   D) 120   E) none of these

13.   The greatest common divisor of two numbers is 30. The least common multiple of the two numbers is 420. The product of the two numbers is:

      A) 210   B) 420   C) 840   D) 1680   E) none of these

14. $2^2 + 2^{-2}$ is equal to:

      A) 0   B) 1   C) $\dfrac{5}{2}$ D) $\dfrac{17}{4}$ E) none of these

15.   The three digit number $2A4$ is added to 329 and gives $5B3$. If $5B3$ is divisible by 3, then the largest possible value of A is:

    A) 4   B) 5   C) 6   D) 7   E) 8

16.   The following figure (not necessarily to scale) consists of two squares with side length of 12 cm and one square with a side length of 8 cm. The area, in cm^2 of the shaded part of the figure is:

    A)   36

    B)   48

    C)   72

    D)   144

    E)   none of these
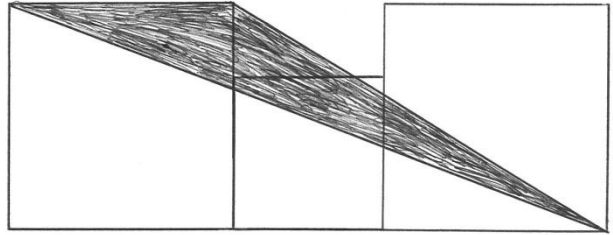
17.   The side, front, and bottom faces of a rectangular prism have areas of 20, 20, and 100 cm^2, respectively. The volume of the prism, in cm^3, is:

    A) 100   B) 200   C) 10,000   D) 400   E) none of these

18.   A basketball tournament has 16 teams. If each team continues to play in the game until beaten once, the number of games needed to determine the champion is:

    A) 5   B) 15   C) 16   D) 17   E) none of these

19.   The midpoint of segment AB has coordinates $(-3,6)$. Point B has coordinates $(5,10)$. The coordinates of point A are:

    A) $(2,18)$ B) $(1,8)$ C) $(-11,2)$ D) $(-8,4)$ E) none of these

20.   The point $(40,-30)$ is reflected in the x-axis. The image is then reflected in the y-axis. The coordinates of the point in its final position are:

    A) $(30,-40)$ B) $(-30,40)$ C) $(-40,30)$ D) $(-40,-30)$ E) none of these

21. Five boxes contain a total of 30 lizards. The first two boxes contain a total of 14 lizards. The second and third boxes contain a total of 10 lizards. The third and fourth boxes contain a total of 9 lizards. The fourth and fifth boxes contain a total of 12 lizards. The number of lizards in the first box is:

A) 9   B) 8   C) 6   D) 5   E) none of these

22. Two circles have diameters PS and QR. If $PS = 3QR$, then the ratio of their areas is:

A) 9 : 1   B) 4 : 1   C) 3 : 1   D) 2 : 1   E) none of these

23. The product of all prime numbers between 1 and 2012 is divided by 21. The remainder is:

A) 0   B) 1   C) 11   D) 17   E) none of these

24. If I toss a fair coin 4 times, the probability that I will get 2 heads and 2 tails (in any order) is:

A) $\dfrac{1}{4}$ B) $\dfrac{3}{8}$ C) $\dfrac{1}{2}$ D) $\dfrac{7}{16}$ E) none of these

25. If $a \times a = 9$, then $a \times a \times a$ could equal:

A) 18   B) 81   C) 729   D) -27   E) none of these

26. If $\dfrac{x}{4} + \dfrac{y}{5} = \dfrac{19}{20}$, where x and y are positive integers, then $x + y$ is:

A) 9   B) 19   C) 20   D) 4   E) none of these

27. If a hen and a half laid an egg and a half in a day and a half, how many eggs would 10 hens lay in a 30-day month?

A) 300   B) 250   C) 200   D) 100   E) none of these

28. If $\frac{1}{3x+3} = \frac{2}{5}$, then $\frac{1}{3x+1}$ is equal to:

   A) 2   B) $\frac{3}{2}$ C) $\frac{2}{3}$ D) $\frac{1}{2}$ E) none of these

29. Of the following sets of angles, which could be the angles of an isosceles triangle?

   A) 150°,10°,20°   B) 110°,40°,40° C) 60°, 60°,60°   D) 90°, 30°,30°   E) none of these

30. In a group of men and women, the average age is 31. If the men's ages average 35 years, and the women's ages average 25 years, the ratio of the number of men to the number of women is:

   A) 5 : 7   B) 7 : 5   C) 2 : 3   D) 3 : 2   E) none of these

31. If I add 5 of the first 6 positive integers, the sum cannot be:

   A) 15   B) 16   C) 18   D) 20   E) 21

32. The sum of five distinct whole numbers is 90. The second largest of these five numbers can be at most:

   A) 40   B) 41   C) 42   D) 43   E) 44

33. If $\frac{2}{3} : \frac{3}{4} = \frac{1}{3} : x$, then x equals:

   A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $\frac{8}{9}$ D) $\frac{3}{2}$ E) none of these

34.   The volume of a sphere is equal to $\frac{4}{3}\pi r^3$ where r is the radius. How many times greater is the volume of the new sphere if the diameter is doubled?

    A) 2   B) 4   C) 6   D) 8   E) none of these

35.   There are 24 4-digit numbers that can be formed, each using all the digits 1, 2, 3, 4. The 5th largest such number is:

    A) 4321   B) 3412   C) 4123   D) 4132   E) none of these

36.   You receive $\frac{2}{5}$ of the $2800 that is owed to you. How much money is still owed to you?

    A) $1120   B) $1680   C) $1780   D) $1880   E) none of these

37.

$$\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}} \quad \text{is equal to:}$$

    A) $\dfrac{3}{5}$ B) $\dfrac{5}{3}$ C) $\dfrac{3}{4}$ D) $\dfrac{4}{3}$ E) none of these

38.   The value of x that will produce the next number in the following geometric sequence is:

$$1000, 500, 250, x - 125$$

    A) 1000   B) 500   C) 250   D) 125   E) none of these

39.   The reciprocal of $\left(\frac{1}{2} + \frac{1}{3}\right)$ is:

     A) 5   B) $\frac{5}{6}$ C) $\frac{6}{5}$ D) 6   E) none of these

40.   The area of a circle whose radius is $\frac{1}{\pi}$ is:

     A) $\frac{1}{\pi}$ B) 1   C) 2   D) π E) none of these