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Teaching Exponents and Logarithms in the Context of Buying a Car

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Introduction

In May 2002, Secretary of the Treasury Paul O'Neill and Secretary of Education Rod Paige convened a panel discussion at the Office of Financial Education (OFE) with leaders of various education and educational testing organizations to discuss how to integrate financial concepts into primary and secondary school curricula (United States Department of the Treasury Office of Financial Education, 2002). The panel recommended that:

- Financial concepts should be included within state standards for math and reading.
- Financial concepts should be included in tests to assess whether students are achieving the levels prescribed by the standards.
- Financial concepts should be integrated into math and reading textbooks.

The state of Indiana requires that students meet standards in financial literacy by the end of the eighth and twelfth grades (Indiana Department of Education, 2011). Schools have flexibility in their delivery of financial literacy, but the Indiana Department of Education encourages them to create separate courses in financial literacy rather than embed those standards into other courses (Ogle, 2012). The advantage of such stand-alone financial courses is the fact that it is easier—from an administrative standpoint—to ensure coverage of all financial literacy standards. The disadvantage, however, is that it is difficult to ensure that students have the mathematical background for calculating compound interest, loan amortization, and other concepts essential to the financial literacy standards. It is worth noting that although financial concepts are not explicitly listed in the Common Core State Standards for Mathematics adopted by Indiana in 2010 (Indiana Department of Education, 2011), they are used in some examples to illustrate the standards.

We recommend integrating financial concepts into mathematics courses to prepare students for financial literacy courses and to add relevance to mathematics. This can be done without loss of instructional time by using financial examples in mathematics courses and by implementing projects involving financial concepts for schools adopting project-based learning.

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The *Indiana Mathematics Teacher* is a peer-reviewed publication of the Indiana Council of Teachers of Mathematics. The *Indiana Mathematics Teacher* provides a forum for mathematics teachers from pre-kindergarten through college to present their ideas, beliefs, and research about mathematics teaching and learning. We are currently seeking manuscript submissions, and welcome them from preK–12 teachers, university mathematics educators, professional development providers, graduate students, and others with a vested interest in mathematics education. Manuscripts should be written for an audience of K–16 mathematics teachers and should be limited to approximately 1500–3000 words. For more information and full submission guidelines see *http://ictm.onefireplace.org/* or contact the editors at *djmohr@usi.edu* and *rhudson@usi.edu*. If you are willing to serve as a peer reviewer to provide feedback on potential articles, contact one of the editors.

In this paper, we present a project-based learning (PBL) unit designed to teach exponents and logarithms in the context of the planning and budgeting it takes to buy a car. PBL is an "extended process of inquiry" for finding a solution to a question that is specifically designed to teach significant content and key concepts of academic disciplines (Buck Institute for Education, 2013). PBL units are organized around a driving question to create a need to learn both essential academic content and 21st Century skills such as critical thinking, problem solving, collaboration, and various types of communication. Mathematical concepts are taught or reviewed as needed in order to maximize students' motivation to learn. PBL projects are carefully planned, managed, and assessed with help from members of the local community; these community members provide instruction of some of the content as well as serve as significant components of the assessment process.

This PBL unit is designed for an Algebra 2 course at Ben Davis University High School (BDU), an early college high school that targets at-risk students and uses PBL—which has been tested in this environment—as one of the main modes of instruction. In the BDU environment, the unit and assessment spanned twelve one-hour class periods. It can be easily adapted for other high school and college mathematics courses where exponents and logarithms are taught. Teaching materials, assessments, and a sample project product are posted on https://carloanmath.pbworks.com.

Unit Overview

High school students want cars because cars give them access to social gatherings, sports, and other activities (McMullen, 2012). This means that a project on car buying has instant relevance. Since an objective of an early college high school is to help students focus on college and career, the driving question for this project combines college focus and buying a car: How can we, as recent college graduates, find the best vehicle for our income?

The unit leads students through the planning and budgeting process so they learn to to buy a car that fits their needs—rather than wants—in terms of lifestyle and affordability. The students, in groups of three or four, apply exponents when analyzing the impact of interest on savings, loans, and depreciation. The information students need for this project is presented throughout a series of in-class workshops interspersed with lab time for the students to do research and develop spreadsheet models for their analyses. Students complete formative assessments to demonstrate their mastery of the mathematics by way of the worksheets they turn in and a practice presentation. At the conclusion of the project, each group organizes a professional presentation of their research with graphs and supporting calculations to explain their choice of a particular automobile. Combined with a traditional test over exponents and logarithms, this becomes a summative assessment as well.

Workshops to Deliver Project Information

The information students need for this project is presented in the professional communication style of corporate seminars and training. A seminar that focuses on a specific topic and lasts one class period is called a workshop. The workshops for this unit include:

1. Car Buying Criteria
2. Car Buying Resources
3. Affordability
4. Compound Interest
5. Depreciation

Car Buying Criteria

At the start of the "Car Buying Criteria" workshop, students imagine their dream car and share their thoughts with the class. They explain what they like about the car, why those characteristics are important, and how those characteristics impact their lives. They also compare their car choices with those of their peers. Next, students work in their groups to develop lists of characteristics that differentiate their "wants" from their needs. Once students have their lists, they watch a short video on purchasing a new car (Reed, 2011) that may bring up items they might not have considered for their lists, such as the expected cost of fuel and maintenance. Students finalize their lists after watching this video.

Car Buying Resources

The purpose of the "Car Buying Resources" workshop is to introduce students to sources of information such as car manufacturers' websites, Consumer Reports (Consumer Reports, 2012), Kelley Blue Book (Kelley Blue Book, 2012), and Edmunds.com (Edmunds.com, 2012). Students are asked to obtain information regarding what percentage of their take-home pay should be allotted for a car payment and why. They are asked to also find webpages that not only calculate the cost for owning cars in terms of fuel, insurance, and maintenance but that also estimate market value for used cars and calculate monthly payments on car loans for various rates of interest and lengths of time. These webpages are then used in the "Affordability" workshop. Next, the students look for cars on the internet that match their list of criteria.

The students augment their list with the following:

- one or more cars that meet the criteria
- a source verifying that the car meets the criteria
- why certain criteria is on their list
- how it affects the value of the car

Finally, students fill out an individual automobile information form with the following data on at least three cars that they might like to purchase:

- Make
- Model
- Year
- Mileage
- 2-wheel or 4-wheel drive
- Automatic or manual transmission
- Purchase price with and without sales tax
- Miles per gallon for both city and highway
- Warranty information
- Kelley Blue Book values for the last five years
- Special features

The process of compiling and augmenting this list is an opportunity to teach students the value of using Word or Excel. It is easy to copy and paste information from websites, including pictures and URLs, into a Word document and make the augmented list into an attractive document to hand in or submit electronically.

Affordability

The "Affordability" workshop begins with students researching how much money a graduate with a degree leading to their planned careers makes at graduation. They estimate the amount of money available to them to spend on a car per month as based on the worksheet shown in Figure 1. Students must understand that this is an estimated budget and that the payments on a vehicle may not be this exact amount. They also need to understand that this amount includes insurance, fuel, maintenance, vehicle registration fees, and other costs. Once students have their monthly car budgets in place, they have an in-class discussion regarding how affordability affects their choice of car.

Next, students compare rates on auto loans from at least three different financial institutions, noting that rates vary by model year of car, term on the loan, whether the seller is a dealer or individual, and whether the interest rate is fixed or variable. Students look up the rates online or call or e-mail the financial institutions.

Based on these rates, students use an auto loan calculator to find the monthly payments for three cars for which they have filled out individual automobile information forms. Auto loan calculators can be found at Autoloancalculator.com (AutoLoanCalculator.com, 2010), Bankrate.com (Bankrate.com, 2012), Edmunds.com (Edmunds.com), and at financial institution websites. The students compare these payments with their monthly car budgets and share their findings with the class.

1. What is your career? ____________________

2. What is the average starting yearly salary? ____________________
   (Total salary before taxes are deducted.)

3. What is your estimated yearly take-home pay? ____________________
   (Taxes and benefits are about 25%.
   Multiply salary by 0.75 to get take-home pay.)

4. What is your estimated monthly take-home pay? ____________________
   (Divide yearly take-home pay by 12.)

5. A conservative budget for your car is 20% of your take-home pay. What is your monthly car budget? ____________________

Figure 1. Worksheet to Estimate Monthly Car Budget.

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Compound Interest

The goal of the "Compound Interest" workshop is for students to understand the impact of compound interest on savings and car loans. They have some background in exponents from previous courses, but review of exponents and an introduction to logarithms is provided as needed. The students explore three situations based on one of their car choices:

1. How much money would I have if I invested the purchase price plus sales tax in a CD at the best rate of interest I can find for the term of the loan?

2. What is the total amount I pay for the car based on the monthly payments?

3. If I put the monthly payments into a savings account now, how long would it take me to accumulate the purchase price plus sales tax for this car? Could I accelerate the process by periodically transferring the amount in the savings account to a CD?

These situations require students to look for the best savings, CD, and loan rates at local financial institutions. The situations are based on the same purchase price plus sales tax so that the effects of interest rates are not hidden by changing the amounts of money.

Suppose a 2009 Honda Accord has a purchase price of $19,998. Including a 7% sales tax, the total price is $21,397.86. A financial institution lists a rate of 3.80% for a 60-month auto loan and a rate of 1.50% for a 60-month CD. For the first situation, a student calculates that an investment of $21,397.86 at 1.50% is worth $23,051.57 at the end of five years using the formula $A=P(1+r)^t$ where A is the amount at the end of time t, P is the principal, r is the interest rate, and t is the time in years. The students demonstrate their knowledge of logarithms by solving the formula $A=P(1+r)^t$ for time and calculating how long it would take to reach various savings goals for different interest rates and principal amounts.

For the second situation, the students learn to use the amortization formula $A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P\left(i + \frac{i}{(1+i)^n - 1}\right)$ to find their monthly payment and check it with an online loan calculator. In this formula, A is the monthly payment, P is the principal, i is the interest rate per time period, and n is the number of time periods. For monthly payments, n=12t where t is the time in years and i=r/12 where r is the annual interest rate. At a rate of 3.80% for a 60-month auto loan, the monthly payment for the 2009 Honda Accord priced at $21,397.86 (including sales tax) is $392.15. Multplying the monthly payment by 60 months gives a total amount paid of $23,528.75. Students are surprised by how much more than the value of the car they pay with a loan.

Having the students develop an amortization table in a spreadsheet that shows how much of each payment is principal and how much is interest (as in Figure 2) really drives the lesson home.

Month	Principal Remaining	Payment	Interest Paid	Principal Paid
1	$21,397.86	$392.15	$67.76	$324.39
2	$21,073.47	$392.15	$66.73	$325.41
3	$20,748.06	$392.15	$65.70	$326.44

Figure 2. First Three Months of Amortization Table.

For the third situation, suppose a student deposits a monthly payment of $392.15 into a savings account with a rate of 0.20%. Students set up the accumulated value in a spreadsheet and determine how many months they would have to save to reach the price of the car. The students may also develop a savings plan to accelerate the accumulation of money by periodically spinning off amounts into a CD at a higher rate of interest.

Depreciation

The goal of the "Depreciation" workshop is for students to compare the Kelley Blue Book values for a given car with a continuous compound interest model for a given depreciation rate and to estimate depreciation rates for the cars they are researching. After an introduction to depreciation, students look up the Kelley Blue Book values for the current year and previous five years of the car they are interested in purchasing, and they plot these values on a graph. On the same graph in Excel, they plot the values from the continuous compounding interest formula, $A = Pe^{rt}$ where A is the amount the car is worth after t years for some depreciation rate r and initial value P. Typically, a car loses 15–20% of its value each year. Figure 3 shows how much a used car purchased for $10,000 is worth for five years of ownership based on both the Kelley Blue Book values and the estimated value from the continuous compounding formula with a 15% depreciation rate. Students estimate the depreciation rates of their cars using three methods

- by experimenting with the depreciation rates on their plot in Excel
- by using logarithms to solve for r in the formula $A = Pe^{rt}$
- by adding the exponential trend line to the Excel plot.

Logarithms are introduced as needed during this workshop.

Year	Estimate	Kelley Blue Book
0	$10,000.00	$10,000.00
1	$8,607.08	$9,758.48
2	$7,408.18	$9,485.75
3	$6,376.28	$8,758.48
4	$5,488.12	$8,061.51
5	$4,723.67	$7,788.78

Figure 3. Depreciation of a Car.

To conclude the workshop, students share what they learn. The example of Figure 3 shows how the Kelley Blue Book value outperforms the depreciation estimate. This is not the case for all cars, and the sharing can show a variety of scenarios. A final question to discuss is: Could you sell the car after one, two, or three years and have enough money to pay back the loan?

Presenting the Research

After they complete all workshops, students assemble all of the information into a Powerpoint presentation and make a decision on the best car. They present their research and decision to an audience of their classmates, school administrators, math instructors at the high school and college level, and financial experts.

The group must include a summary table of decision criteria on which to base a purchase with candidate cars as shown in Figure 4. Groups must have a minimum of three cars and a minimum of four decision criteria. Their decision criteria must include:

1. purchase price with monthly payments

2. resale value

3. lifestyle fit

The fourth criterion is left up to group choice. Additional criteria can improve projects if they are well-researched.

Combining the criteria for a decision requires critical thinking. Although the first two criteria add in a way that makes sense, combining all four criteria numerically does not. The simplest approach is to select the best car for each criterion and sum the number of bests. Students can create more sophisticated scoring systems based on their personal preferences, but they need to understand that different scoring schemes can give different answers while there also exist many possible correct answers.

Unit Evaluation

This instructional unit was evaluated based upon:

1. Student achievement on a summative test over exponents and logarithms

2. Evaluations of student presentations

3. A questionnaire filled out by students on their perceptions of the impact and relevance of the project.

The test results indicate that 68% of the students achieved proficiency and 26% achieved high proficiency. The difference between this year's average score and the averages of equivalent tests on exponents and logarithms taught in the traditional way in Algebra 2 at Ben Davis University High School over the last three years is not statistically significant. Students struggled with the math somewhat but did not struggle any more than they do in a traditional format.

The aspect of the project with which students struggled the most was the presentation. Having an extra peer evaluation day where the students presented to another group and received feedback helped them to refine their presentations and gave them more confidence in

Criterion	2005 BMW 325	2009 Honda Accord	2008 MINI Cooper
Purchase price + 7% tax Monthly payments, 60 mo	$18,187.86 $347.41 @ 5.5%	$21,397.86 $392.15 @ 3.8%	$20,970.93 $387.16 @ 4.1%
Fuel costs for 1000 mi/mo @ $3.50/gal	18/26 MPG $159.09	21/30 MPG $137.26	26/34 MPG $116.67
Resale value	About 15% loss per year	Less than 15% loss per year	Less than 15% loss per year
Does the car fit my life?	Convertible is cool	4-door for a family	Hatchback for loads

Figure 4. Decision Criteria Summary.

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their final presentation in front of the panel of community members. The average score on the presentations given by the panel comprised of school administrators, high school and college level math instructors, and financial experts was a 90%. Other audience members also made many positive comments. Another math teacher commented that the project "was extremely engaging for our high school students ready to step into their next phase of life. They were challenged with the mathematical content of compound interest, displaying their collected data graphically, and comparing various scenarios. The students found great value in the project as they realized they would be in the market for a car purchase very soon if not already. The presentation was a great experience for the students as they had to provide the rationale and support for their decision. Overall, the PBL unit was practical, engaging, and supplied a healthy dose of meaningful content for our BDU early college high school students." On the questionnaire, 91% of students reported that they found the math in the project to be relevant, and 86% indicated that this project influenced how they would choose a car in the future. One of the students commented, "Learning math like this was hard, but I think it was good. I think I understand it better because I can think about the project and remember how to do it." Another summed up the experience by saying, "I was way more interested in the math because I was buying my dream car."

Conclusion

This PBL unit demonstrates how integrating financial concepts into math can be done without loss of quality of mathematics instruction and with significant gains in relevancy. By combining the mathematical content of exponents and logarithms with the practical financial applications of purchasing a vehicle, students can make strong connections with the material that will serve them long after they have completed their high school courses. Such a project allows students more than just an opportunity to become familiar with the mathematical and financial concepts; it ultimately provides them with an opportunity to learn real life skills with hands-on experience and within a safe practice environment.

Students are encouraged throughout the project to think deeply about the mathematical material and to apply it to a situation which is highly personalized as it is based on their own preferences and personal aspirations. This high level of personalization then results in students' high levels of engagement with the material. The inherent practicality of learning how to navigate the car buying process is also engaging for students; and, in the end, they progress in terms of their experience with financial vocabulary, mathematics, computer applications, internet resources, and general presentation skills. Students are able to display well-rounded growth rather than the rote understanding of a concept.

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Comparing Indiana and National Mathematics Performance

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Along with 44 other states, Indiana is transitioning to the use of the Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010). Piloting of assessments tied to these standards has begun in the 2012–2013 school year for the 24 states that have joined the Partnership for Assessment of Readiness for College and Careers (PARCC) consortium. These new assessments will then replace Indiana's current ISTEP+ mathematics assessments in the 2014-2015 school year.

One of the stated purposes for the PARCC assessments is to allow for comparisons among states (see, for example, Thompson and Mirabelli, 2011). Some comparisons among states, however, are possible now using National Assessment of Educational Progress (NAEP) results. NAEP uses representative samples for the nation and states to assess student performance in a number of content areas. Therefore, we sought to compare Indiana's performance on NAEP to the performance of the United States as a whole. In particular, we looked for differences in the types of items on which Indiana's students outperformed the nation.

Indiana has outperformed the national average on NAEP mathematics assessments for all but one administration in grade 4 and in all administrations in grade 8 since 1992 (National Center for Educational Statistics, 2011a). In 2011, only six states performed significantly higher than Indiana, and 26 states performed significantly lower (National Center for Educational Statistics, 2011a).

Of course, the performance of Indiana students has varied across the individual NAEP items. An analysis of item-level statistics for items released in 2005, 2007, 2009, and 2011 revealed that the percentage of Indiana students correctly answering the majority of items was not significantly different from the percentage of students correctly answering in the national sample. For a limited number of items, however, a significantly higher percentage of Indiana students answered correctly. Indiana grade 4 students performed significantly better than the nation on 24 items, and Indiana grade 8 students performed significantly better than the nation on 19 items (for more detailed information about these items, see Appendices A and B, available online at http://www.indianamath.org).

The NAEP mathematics assessment is developed using a framework that describes the types of questions that should be included and how they should be designed and scored. Beginning in 2005, items on the mathematics assessment were identified by complexity level as shown in Figure 1.

Low Complexity
This category relies heavily on the recall and recognition of previously learned concepts and principles. Items typically specify what the student is to do, which is often to carry out some procedure that can be performed mechanically. It is not left to the student to come up with an original method or solution.

Moderate Complexity
Items in the moderate-complexity category involve more flexibility of thinking and choice among alternatives than do those in the low-complexity category. They require a response that goes beyond the habitual, is not specified, and ordinarily has more than a single step. The student is expected to decide what to do using informal methods of reasoning and problem-solving strategies and to bring together skill and knowledge from various domains.

High Complexity
High-complexity items make heavy demands on students, who must engage in more abstract reasoning, planning, analysis, judgment, and creative thought. A satisfactory response to the item requires that the student think in an abstract and sophisticated way.

Figure 1. NAEP Levels of Complexity. From Mathematics Framework for the 2009 National Assessment of Educational Progress by the National Assessment Governing Board, 2008, Washington, DC: Author, p. 37.

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Comparing Indiana and National Mathematics Performance *Continued...*

The moderate and high levels of complexity share many characteristics with the Common Core Standards for Mathematical Practice (Figure 2). NAEP's description of moderate-complexity items includes an emphasis on flexible thinking, which is a quality also featured in the Common Core math practices involving reasoning, modeling, strategic use of tools, and making use of mathematical structure. Similarly, NAEP's description of high-complexity items includes an emphasis on abstraction and analysis—two ideas addressed in the Common Core math practices involving reasoning and constructing viable arguments. NAEP's descriptions of moderate- and high-complexity items do not directly address some aspects of the Standards for Mathematical Practice, such as perseverance, but significant alignment and overlap do exist between the two.

In addition to complexity level, each NAEP item is identified as belonging to one of three difficulty levels (easy, medium, or hard) based on the percentage of responses scored as correct. Items are designated easy if 60% or more of students responded correctly, medium if 40% to 59% responded correctly, and hard if less than 40% responded correctly (National Center for Educational Statistics, 2011b). Figure 3 provides examples of NAEP items at different levels of difficulty and complexity.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Figure 2. Standards for Mathematical Practice. From Common Core State Standards Initiative (2010). Common Core State Standards for Mathematics by the Common Core State Standards Initiative, 2010, Washington, DC: The National Governors Association Center for Best Practices and the Council of Chief State School Officers, pp. 6-8.

Easy difficulty, low complexity items

Grade 4 (2011-4M9 #13)

How are the right triangle and the rectangle alike?

A. Each figure has at least one right angle.

B. Each figure has parallel sides.

C. Each figure has at least one line of symmetry.

D. Each figure has at least two sides that are the same length.

Grade 8 (2009-8M10 #4)

In which of the following numbers is the digit 6 in the hundredths place?

A. 682.3

B. 382.6

C. 6.832

D. 4.836

E. 2.862

Hard difficulty, high complexity item

Grade 4 (2011-4M12 #11)

A student had to multiply 328 × 41. The student's answer was 4,598.

Use estimation to explain why this answer is not reasonable.

Hard difficulty, moderate complexity item

Grade 8 (2005-8M4 #18)

3 pineapples

1 serving = ½ pineapples

Given the information above, write a mathematics word problem for which 3 ÷ ½ would be the method of solution.

Figure 3. Examples of NAEP items at various difficulty and complexity levels. Adapted from the NAEP Questions Tool by the National Center for Educational Statistics, 2011b, retrieved from http://nces.ed.gov/nationsreportcard/itmrlsx.

Table 1 provides a summary of the difficulty and complexity levels for those items on which Indiana students performed significantly higher than the nation's average. In grade 4, 63% of these items were of easy difficulty and 75% of low complexity; and, in grade 8, 74% of these items were of easy difficulty and 79% of low complexity. These easy and low-complexity items focused mainly on procedural knowledge rather than the higher-level cognitive processes and proficiencies suggested by the Standards for Mathematical Practice.

Table 1. Difficulty and Complexity Levels for Grades 4 and 8 Items on Which Indiana's Performance was Significantly Greater ($p < 0.01$)

	Difficulty				Complexity		
	Easy	Medium	Hard		Low	Moderate	High
				Grade 4			
2005	2	1	0		3	0	0
2007	7	5	1		8	5	0
2009	3	0	0		3	0	0
2011	3	2	0		4	1	0
Overall	15	8	1		18	6	0
				Grade 8			
2005	4	1	0		4	1	0
2007	2	2	0		4	0	0
2009	6	1	0		6	1	0
2011	2	1	0		1	2	0
Overall	14	5	0		15	4	0

The released items do include more easy items than medium and hard items, and they do include more low-complexity items than moderate- and high-complexity items. For the grade 4 released items studied, 43% were easy, 34% medium, and 23% hard; for grade 8, 41% were easy, 33% medium, and 27% hard. For the grade 4 released items studied, 65% were low complexity, 32% moderate complexity, and 3% high complexity; for grade 8, 59% were low complexity, 39% moderate complexity, and 1% high complexity. However, easy, low-complexity items were still overrepresented among the items on which Indiana outperformed the nation.

Indiana's performance was significantly lower than the nation on a smaller number of items—only seven for grade 4 and three for grade 8. Table 2 offers a summary of the difficulty and complexity levels for these particular items (for more detailed information about these items, see Appendix C, available online at http://www.indianamath.org). All but one of these items fell into the medium or hard levels of difficulty, but they were all represented among the three levels of complexity.

Table 2. Difficulty and Complexity Level for Grades 4 and 8 Items on Which Indiana's Performance was Significantly Lower ($p \leq 0.01$)

	Difficulty				Complexity		
	Easy	Medium	Hard		Low	Moderate	High
				Grade 4			
2005	0	1	0		1	0	0
2007	0	0	0		0	0	0
2009	1	1	1		1	2	0
2011	0	2	1		1	0	2
Overall	1	4	2		3	2	2
				Grade 8			
2005	0	0	1		0	1	0
2007	0	0	0		0	0	0
2009	0	0	0		0	0	0
2011	0	1	1		2	0	0
Overall	0	1	2		2	1	0

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Comparing Indiana and National Mathematics Performance *Continued...*

Released NAEP items are not necessarily representative of the entire set of NAEP items. Still, these results clearly suggest that while Indiana students are generally above the national average on NAEP, their strengths are more prominent in the area of procedural skills rather than on tasks that require complex mathematical thinking.

These results may have implications for Indiana's implementation of the Common Core State Standards for Mathematics. Schmidt (2012) judged Indiana's standards—as well as the standards of nine other states—to be the existing state standards best aligned to the Common Core. This judgment reflects a relatively strong similarity between the content of Common Core and Indiana's current standards. This would appear to give Indiana an advantage in implementing Common Core. However, the emphasis on conceptual understanding in the new Common Core will lead to items that are more complex in nature that will require more than the use of procedural skills. In addition, items addressing the Standards for Mathematical Practice will tend to be more difficult and require abstract reasoning, which is a characteristic of the current high-complexity items. Since Indiana's higher performance on NAEP is largely due to students' better results on procedural items, Indiana appears to be no better prepared than the nation as a whole in meeting the demands of the Common Core State Standards for Mathematics. With that being said, we believe the challenge of implementing the new standards—including the Standards for Mathematical Practice—must be met and embraced if we are to prepare Indiana's students for careers that are increasingly demanding higher-level reasoning skills, complex analysis, and creative thought. Focusing on the mathematical practices needs to be a key part of the transition to the Common Core.

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