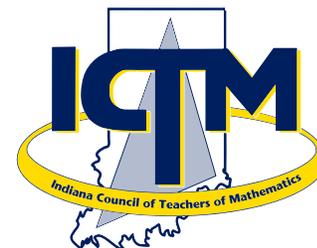


# Indiana Mathematics Teacher

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## College Credit for High School Calculus

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Calculus I is a foundational course for all students wishing to major in mathematics or engineering, and it is a gateway course for all students pursuing a major in one of the hard sciences such as chemistry, biology, or physics. In previous years, students have generally earned college credit for a high school calculus course by surpassing a benchmark score on a nationally-normed test (Bressoud, 2009). This scenario, however, has changed as dual enrollment—a second option for earning college credit—has moved to the forefront. Nearly 15,000 public high schools enrolled students in 2 million dual-enrollment college courses during the 2010-11 school year (NACEP, 2013). Many stakeholders perceive these courses to be beneficial. Students like them because they can earn college credit without the pressure of a high-stakes assessment such as the AP exam; parents like them because of the lower tuition rates and the potential to shorten their child's pathway to degree completion; high school personnel like them because of the prestige associated with courses bearing college credit; and, college administrators like them because they view the dual-enrollment courses as both a revenue stream and recruitment tool (Bressoud, 2007). Additionally, state legislatures pressure schools to offer dual-enrollment courses. The state of Indiana requires high schools to offer students at least two dual-enrollment courses (Indiana Commission for Higher Education, 2010).

The University of Southern Indiana (USI) is a National Alliance of Concurrent Enrollment Partnerships (NACEP)-accredited dual-enrollment program that partners with local high schools at the college algebra and trigonometry levels. We chose not to participate at the calculus level. However, other colleges and universities award college credit for high school calculus, and many of these students will enter our university with this credit. This paper investigates the mathematical profiles of students entering our university after successfully completing a high school calculus course. The purpose for this study was two-fold: we were looking for data to either support or refute that there is no difference between the two predominant methods for awarding college credit to students who successfully complete two semesters of high school calculus; and, we were also looking for data indicating the level of algebraic proficiency to be expected from students completing two semesters of high school calculus.

### Data Collection

During the fall 2009 semester, we asked the following questions to all students enrolled in a math course up to the level of Calculus II: "Did you successfully complete a calculus course—not a pre-calculus course—in high school?"

*Continued...*

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### About the Journal

The *Indiana Mathematics Teacher* is a peer-reviewed publication of the Indiana Council of Teachers of Mathematics. The *Indiana Mathematics Teacher* provides a forum for mathematics teachers from pre-kindergarten through college to present their ideas, beliefs, and research about mathematics teaching and learning. We are currently seeking manuscript submissions, and welcome them from preK-12 teachers, university mathematics educators, professional development providers, graduate students, and others with a vested

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interest in mathematics education. Manuscripts should be written for an audience of K-16 mathematics teachers and should be limited to approximately 1500-3000 words. For more information and full submission guidelines see <http://ictm.onefireplace.org/> or contact the editors at [djmohr@usi.edu](mailto:djmohr@usi.edu) and [rhudson@usi.edu](mailto:rhudson@usi.edu). If you are willing to serve as a peer reviewer to provide feedback on potential articles, contact one of the editors.

# College Credit for High School Calculus Continued...

Every 'yes' response was verified by checking the students' high school transcripts. We included in this study only first-time, full-time students (a total of 133 students) who earned a grade of C or better in both semesters of high school calculus. Each of these 133 students was matched with additional available data, including their USI math placement (described below), their AP exam score, and their final grade in Calculus I at USI. Students with high school calculus who did not enroll in a USI math course were not represented in our data. The decision to use data from the fall 2009 semester was based on the fact that it was the last semester prior to an expected increase in the number of students entering our university with college credit for high school calculus.

## Data Summary and Analysis

Of the 133 students in this study, 68 students took the ACCUPLACER test and were given a USI math placement; 53 students had AP exam scores; and, 45 students enrolled in Calculus I at our university. See Figure 1 for distribution of the 133 students into these three subsets.

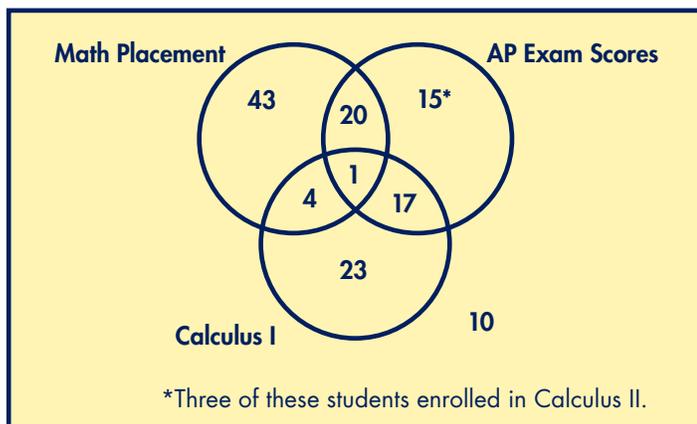


Figure 1. Distribution of Students Within the Study

The first subset we investigated consisted of students who were given a USI math placement. Freshman entering our university are required to take the ACCUPLACER math placement test if their SAT/ACT math score is below 600/26. This subset consists of students who tested within the lower SAT/ACT scores (average 539/23); hence, it is not representative of the entire cohort. A previous study showed that math placement results were significantly improved by including students' SAT/ACT math scores and high school percentiles in the placement model (Rodgers and Wilding, 1998). As detailed in Table 1, 91% of students in this subset were assigned placements below the level of Calculus I using the placement model. In addition, 65% of students who successfully completed a high school calculus course placed below the level of pre-calculus. Only six students (8%) earned placement scores indicating readiness for Calculus I.

Table 1. Number of Students Placing into Each Course Group

Placement	Frequency
Developmental	7
College Algebra	37
Pre-Calculus	18
Calculus I	6
<b>Total</b>	<b>68</b>

The second subset we analyzed consisted of the 53 students (from the total cohort of 133) with AP scores (AB level). All but four of the 53 students in this subset were enrolled in a high school calculus course with AP designation. These four students had AP scores of 1 and are not included in the analysis of this subset of students as described in Table 2. A high percentage (71%) of students earned the lowest possible score on the AP exam. If these students had successfully completed calculus through a dual-enrollment program, all of them would have earned college credit for the first course in calculus. However, even the most liberal interpretation of AP scores would have awarded college credit to only seven (14%) of these students. Verifying whether or not these students were enrolled in dual-enrollment courses was not possible, but it is reasonable to believe that a high percentage of high school AP Calculus programs would qualify for dual-enrollment. This assumption is based partially on the fact that the topics covered in an AP Calculus course (CollegeBoard AP) closely align with those of a typical university calculus course. This assumption has also been validated by discussions with the department chair of a major provider of college credit for dual-enrollment calculus courses who indicated that AP Calculus courses were generally accepted for dual enrollment provided that both the credentials of the high school instructor were satisfactory and the students either took the AP exam or the college's final exam for that course. If these students had been in a dual-enrollment course, the college grade could have been conceivably different from the verified high-school grade. It is from our experience in evaluating transcripts that we rarely find a grade for the college course differing from that on the high school transcript. For these reasons, we acknowledge the possibility that some of the students in this subset may not have earned college credit for their AP Calculus course; however, it is our professional opinion that even accounting for these two possibilities would not have reconciled the inconsistency in the two predominant methods for awarding college credit for a high school calculus course for this particular subset of students. And, even though the sample size is small, it should be noted that all students who earned a score of 4 or 5 on the AP Exam and enrolled in Calculus II were successful.

Table 2. Advanced Placement Scores from Students Enrolled in AP Calculus

Score	Frequency
5	4*
4	0
3	3
2	7
1	35
Total	49

\*Three of these students enrolled in Calculus II; all were successful. The fourth student received credit for Calculus I but still enrolled in a business calculus course.

The third subset we analyzed consisted of the 45 students (from the total cohort of 133) who enrolled in Calculus I. The average SAT/ACT math score was 629/28 for this subset of students. As listed in Table 3, 71% percent of these students earned a C or better in their first university calculus course. These students had a higher success rate than the general population (54%) of Calculus I students in the fall of 2009, which suggests that successfully completing two semesters of high school calculus improves success rates in a subsequent university Calculus I course.

Table 3. Grade Distribution of Calculus I Students

Grade	Frequency
A	14
B/B+	9
C/C+	9
D/D+	5
F	6
W	2
Total	45

## Conclusions

By investigating the mathematical profiles of students successfully completing two semesters of high school calculus, it was possible to assess the consistency between the two predominant methods of awarding college credit for high school calculus. Even though we could not verify the percentage of students in our study who participated in a dual-enrollment course, it is likely that the 49 students who took both the AP Calculus course and the AP exam would have qualified for dual enrollment since the AP designation for a high school calculus course is a major determining factor for dual enrollment qualification. We believe, as such, that there exist implications from this AP exam data in regard to dual-enrollment courses. Presumably then, one method for awarding college credit would have awarded credit to 49 students, and the other method would have awarded college credit to—at most—seven of these students. Even if only half of the 49 students received college credit, there would still exist a substantial difference. This data suggests a disparity in the two methods of awarding college credit: a greater percentage of students earn college credit for a high school calculus course through dual enrollment than through AP exams.

This study also investigated the level of algebraic proficiency to be expected from students completing two semesters of high school calculus. From the cohort of 133 students, 62 of the 68 who took the placement test placed into a course below the level of calculus—despite having successfully completed calculus in high school. Since the USI math placement is based primarily on measures assessing algebraic proficiency, this data suggests that 91% of the students in this cohort did not have the algebraic proficiency level required for success in Calculus I. A natural question to pose at this point is, “Is algebraic proficiency being short-changed in the rush to get students to calculus?” Placing into college algebra after completing a high school calculus course is reason for concern to us and others within the field of mathematics education. The joint position of the Mathematical Association of America and the National Council of Teachers of Mathematics (2012, p.1) states, “Students who enroll in a calculus course in secondary school should have demonstrated mastery of algebra, geometry, trigonometry, and coordinate geometry...” Our study does not provide sufficient data to determine whether or not students’ algebraic proficiency is being short-changed, but we encourage exploring the question and discussing the findings. Conversations among high school and college faculty examining algebraic proficiency could provide additional insight into and the possible identification of other factors impacting algebraic proficiency.

**Continued...**

## Implications

A first course in calculus may or may not be a terminal math course for students. When Calculus I is the terminal course, earning credit without the accompanying knowledge is problematic since students may enroll in other courses where there is a presumed level of knowledge of college algebra and calculus. Our data does not support these presumptions. When Calculus I is not the terminal course, students must enroll in a second course in calculus. If you agree with the premise that a score of 1 or 2 on the AP exam indicates a lack of preparation for Calculus II, then our data strongly suggests that this cohort of students was not prepared for a second course in calculus.

Our study also indicates a lack of algebraic proficiency for many students. Deficiency in these skills is problematic to students in a variety of disciplines including chemistry, physics, biology, geology, economics, math for elementary majors, finance, marketing, and kinesiology among others. Unfortunately, this is a problem that has been exacerbated by the systemic issue of pushing students through to courses regardless of their developmental readiness.

Our overall data indicates that the two most common methods for offering credit for high school calculus—dual-enrollment courses and AP exams—are not consistent. Could part of the disparity possibly be corrected through closer scrutiny of dual-enrollment courses? NACEP has established procedures for the accreditation of

higher-education programs offering dual-enrollment programs. This accreditation process includes assessment of curriculum as well as evaluation standards (Laurent, 2009); however, only a small number of higher-learning institutions are accredited through this agency. As of April 2012, only 83 dual-enrollment partnerships throughout the United States were accredited through NACEP (NACEP, 2012). This lack of accountability has the potential to cause later problems for both students and the colleges who admit them (Bressoud, 2007).

Although accreditation would likely improve dual-enrollment program offerings, we suggest that policies could be put in place to promote the consistency of the academic rigor of these college-credit courses taken at the high school level. These policies might include the following: placement testing in the high school prior to enrolling in a dual-enrollment course; administering the same final exam as given on the college campus; weighting the final exam to count for at least 20% of students' final grades; eliminating the policy that the high school grade and college grade be the same.

We do not have ready solutions for the issues discussed in this paper, but we do know that future discussions need to take place since encouraging students to take high school calculus for college credit may not always be in their best interest.

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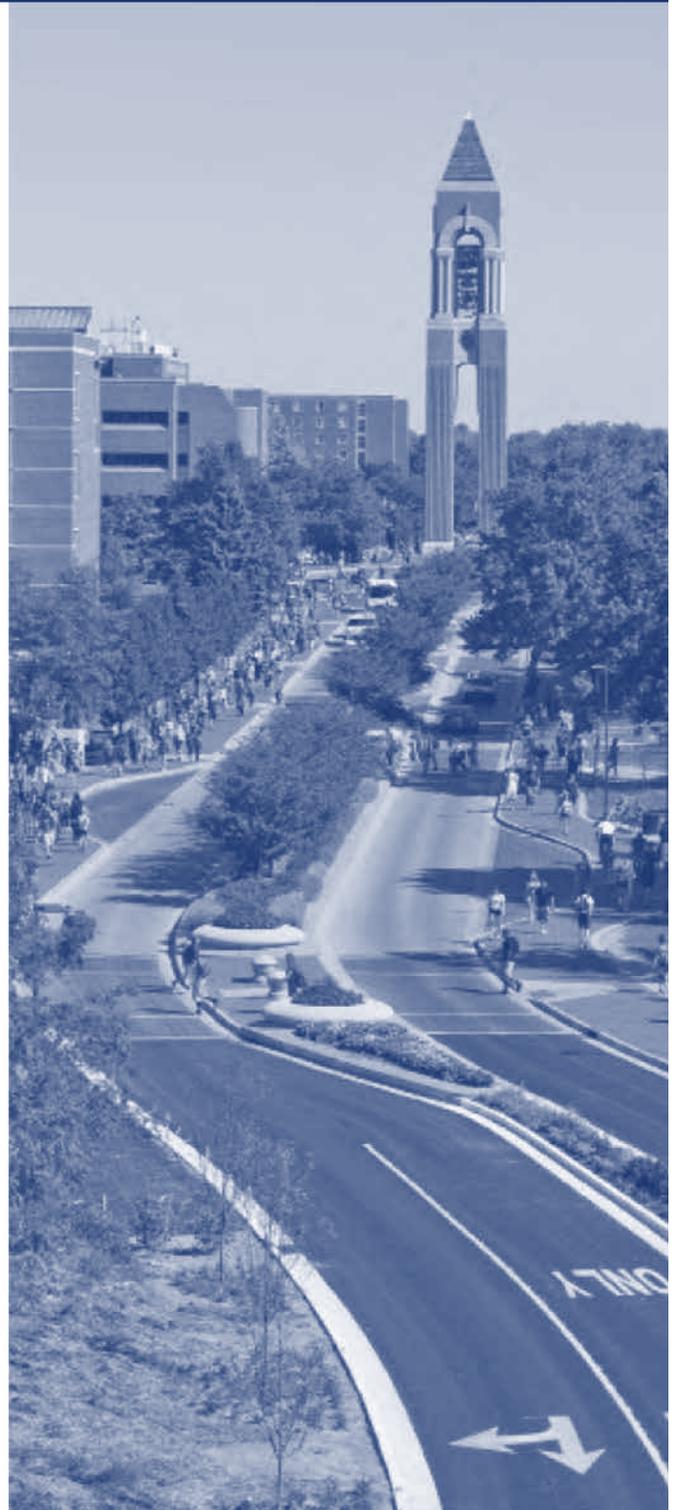
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# How Many Tables? Increasing Cognitive Demand While Incorporating Mathematical Practices

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Cory A. Bennett, Idaho State University

“Different tasks require different levels and kinds of student thinking,” and sustaining the challenge of rich tasks can be complex as students begin to work on problems (Stein, Smith, Henningsen, & Silver, 2009, xviii). Recent research has shown that when teachers are cognizant of the cognitive demand of tasks, they are able to more frequently select tasks to appropriately challenge their students while maintaining high levels of cognitive engagement throughout the implementation of the task (Boston & Smith, 2009; Jackson et al., 2013). Tasks with high cognitive demand require complex, non-algorithmic thinking and engage students in exploring and understanding concepts, processes, or relationships (Stein et al., 2009). In addition, these tasks require that students self-monitor their progress, access relevant knowledge, and engage in solving problems with unpredictable processes—all of which are fundamental elements of meaningful problem solving. At times, it can be difficult to distinguish tasks that demand high levels of cognition from those that require lower to medium levels of demand. Simply using manipulatives, or making a connection to a real world context does not deem a task to be cognitively demanding. Likewise, understanding that the mathematical difficulty of a task is not synonymous with the cognitive demand of a task can be challenging.

## Overview of Mathematical Tasks Framework

Stein et al. (2009) created ‘The Task Analysis Guide,’ a framework that provides characteristics for four levels of cognitive demand: ‘Memorization’ tasks, ‘Procedures *without* Connections’ tasks, ‘Procedures *with* Connections’ tasks, and ‘Doing Mathematics’ tasks. ‘Memorization’ tasks and ‘Procedures without Connections’ tasks are considered to be lower-level tasks. ‘Memorization’ tasks involve the reproduction of already-learned facts, lack connection to concepts, and cannot be solved with procedures because procedures do not exist. An example of a ‘Memorization’ task would be: “Solve the following four facts,  $4 \times 9$ ,  $3 \times 8$ ,  $2 \times 5$ ,  $1 \times 7$ .” ‘Procedures without Connections’ tasks are those that are algorithmic, use procedures, lack connection to concepts, and are focused on producing correct answers. An example of a ‘Procedures without Connections’ Task would be: “Solve  $4x - x = 15$ .” Of higher cognitive demand are tasks termed ‘Procedures with Connection’ tasks. These are distinguished from ‘Procedures without Connections’ tasks in that they are represented in multiple ways, focus on using procedures to develop deeper understanding, require cognitive effort, and have a connection to underlying concepts. An example of a ‘Procedures with Connections’ task would be: “Determine what is  $\frac{1}{3}$  of  $\frac{3}{4}$  and make two different representations to support your solution.” Problems with the highest level of cognitive demand are termed ‘Doing Mathematics’ tasks.

These tasks require complex thinking in which students explore and understand relationships while self-monitoring their mathematical understanding. ‘Doing Mathematics’ tasks require that students analyze the tasks and exude cognitive effort, and the tasks often provoke anxiety in students (Stein et al., 2009). An example of a ‘Doing Mathematics’ task would be: “Of 6000 apples harvested, every third apple was too small (s), every fourth apple was too green (g), and every tenth apple was bruised (b). The remaining apples were perfect (p). How many perfect apples were harvested?” Figure 1 provides the trajectory of the four types of tasks.

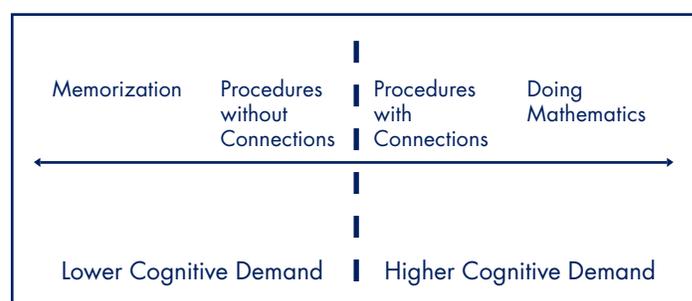


Figure 1. Levels of cognitive demand based on Stein, et al. (2009)

## Teachers in Action

In a recent professional development session with a group of kindergarten, first-, and second-grade teachers, the importance of cognitively demanding tasks arose as a topic for discussion. Teachers were questioning whether tasks at the primary grades could be cognitively demanding because they considered the mathematics content to be too basic or simplistic. As the teachers engaged in a process of working with tasks, modifying tasks, and considering both the tasks’ relationship to the Standards for Mathematical Practice (SMP) (Common Core State Standards for Mathematics, 2010) and the students’ behaviors demonstrated while engaging in mathematics, they realized they could setup, introduce, and sustain the cognitive demand of tasks in their primary classrooms. Additionally, they were able to align higher cognitive demand tasks with instructional strategies aligned with the SMPs, thus improving their plans for teaching. Knowing how to modify tasks and incorporate the SMPs when planning is important for providing students with opportunities to grow and develop mathematically (Boston & Smtih, 2009).

## Increasing Cognitive Demand

To begin working on the modification of mathematical tasks, the participating teachers were presented with tasks from four curricula series. They were then charged with analyzing each task and

determining the level of cognitive demand of the problems. One second grade problem read, "There are square cafeteria tables at Crowley Elementary School. Four children can sit at each table. Show how many tables Mrs. Kavanaugh will need for the 23 children in her class." (Saxon, 2013). The teachers used 'The Task Analysis Guide' (Stein et al., 2009) to analyze the task and determined it was a 'Procedures without Connections' task because it required students to produce correct answers but did not require an explanation that extended beyond a description of the procedures used. The teachers argued that the task was not 'Procedures with Connections' because it did not require students to engage in understanding the underlying concepts; students could create equal groups of four until all twenty-three children were accounted for. It should be noted that by bringing attention to the relationship of creating equal groups, this task has potential to be 'Procedures with Connections' as teachers can also focus on developing students' conceptual understanding of division. Of particular interest to these teachers, this curriculum series included this problem in a section termed "Draw a Picture" but did not create a context for students to represent their work in multiple ways. After determining that the cognitive demand of the task was 'Procedures without Connections,' the teachers sought to increase the level of demand. As a result, they worked together to modify the task. Figure 2 shows how one teacher, Mr. Barton, began to modify the original task.

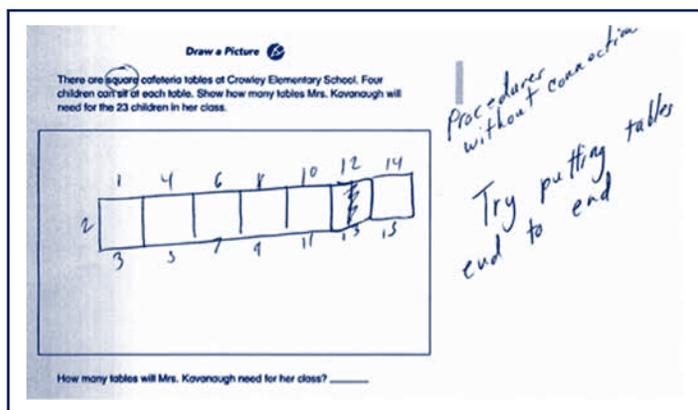


Figure 2. Mr. Barton's ideas to increase the cognitive demand of the task

As Mr. Barton worked with two other teachers to modify the task, they began to formulate the words they would use to extend the task. This resulted in the creation of the task in Figure 3. The top portion of the chart paper is the original task and the bottom portion represents the additional language added to increase the cognitive demand of the task. This process helped the teachers realize they can take the curricula series they are using in their classrooms and make minor modifications to the problems to increase the cognitive demand of the tasks, thus improving the learning experiences they provide for students.

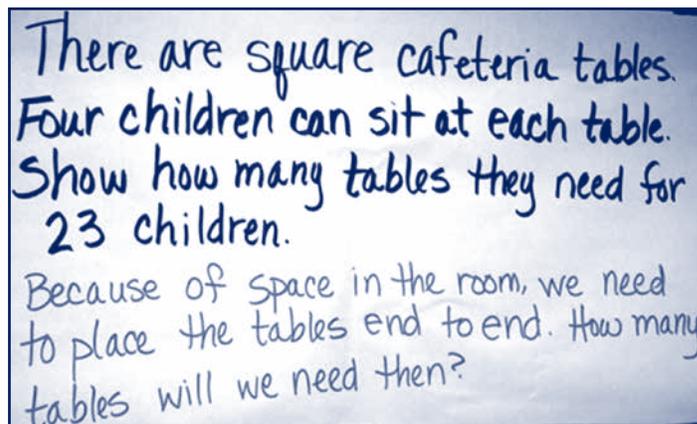


Figure 3. Original and modified version of the task to increase the cognitive demand of the task

## Linking Tasks and the SMPs

After modifying their tasks, teachers were charged with thinking about how they would maintain the cognitive demand of the task as they implemented the task in their classrooms. The group felt that considering the tenets of the CCSS-M Standards for Mathematical Practice (SMPs) could help them think about important instructional strategies to consider as they strive to maintain the cognitive demand of the tasks. Given that the SMPs grew out of the NCTM process standards (NCTM, 2000) of problem solving, reasoning and proof, communication, representation, and connections as well as out of the strands of mathematical proficiency specified in the National Research Council's report "Adding It Up" (Kilpatrick, Swafford, & Findell, 2001), the teachers believed that engaging in discussions around potential alignments between the SMPs and the modified tasks would increase both the probability of maintaining the cognitive demand of the task as well as the likelihood of their students developing the habits of mind outlined in the SMPs. To facilitate this thinking, they engaged in discussions about the eight SMPs (CCSSM, 2010):

1. Make sense of problems and persevere in solving them.
  2. Reason abstractly and quantitatively.
  3. Construct viable arguments and critique the reasoning of others.
  4. Model with mathematics.
  5. Use appropriate tools strategically.
  6. Attend to precision.
  7. Look for and make use of structure.
  8. Look for and express regularity in repeated reasoning.
- (p. 6-8).

**Continued...**

## How Many Tables? *Continued...*

Each teacher then purposefully selected one SMP for focus. Their rationales for choosing their respective SMPs varied, but many chose an SMP they felt either related to the chosen task or wished to better develop with their own students. Even though the potential to embed the practices within multiple SMPs would likely arise as a result of students engaging in the tasks, these teachers considered just one, so that they could deliberately focus their planning and thinking around supporting the premise of their particular SMP in the instructional strategies they outlined. Teachers then began to consider how they would implement the modified problem while focusing on their chosen SMP and identifying instructional strategies to maintain the level of cognitive demand. To aid in these considerations, teachers were asked to consider the cause and effect of their planned instruction—with the effect being implications for learning opportunities and the cause being implications for teaching. The teachers were asked to consider, “If this is what I want to see my students doing (effect)... then this is what I need to plan for instruction (cause).” Considering both the cause and effect of their actions and focusing on the one problem they modified with their chosen SMP required teachers to be explicit about their instruction and the exact means by which they would engage students in learning mathematics. One teacher, Mrs. Appleby, focused on SMP #5, “Use appropriate tools strategically.” As she considered the cause and effect of her instruction, she thought about specific ways she would scaffold opportunities for learning by using manipulatives as the students solved the modified table problem. Figure 4 shows Mrs. Appleby’s beginning thoughts regarding her plans.

Effect: Implications for learning opportunities If this is what I want to see my students doing...	Cause: Implications for teaching ...then this is what I need to plan for instruction.
<p>I would like to see my students use appropriate tools strategically as they solve a problem on table placement &amp; providing enough tables for a given amount of students.</p> <p>Can we rearrange the room to <del>fit</del> a horseshoe design with the same amount of tables?</p>	<p>As I plan for this lesson I need to give students multiple <del>&amp;</del> manipulatives and experiences (from previous lessons) to be able to solve this problem</p> <p style="text-align: right;"><i>sea field</i></p>

Figure 4. Mrs. Appleby’s cause and effect sheet as she considered how to set up and teach the problem about the tables

It is interesting to note that as Mrs. Appleby thought about her problem, she anticipated that students may consider other possibilities for arranging the tables. She writes, “Can we rearrange the room to a horseshoe design with the same amount of tables?” As Mrs. Appleby engaged in this process, she was aware of other tangents for the lesson and considered extensions that would help students learn. In essence, by selecting and modifying a task to increase the cognitive demand and by deliberately mapping the expected student outcomes to instructional actions, Mrs. Appleby became aware of how rich tasks naturally lend themselves to differentiation.

As they worked on modifying different tasks and engaged in considering the cause and effect of their actions, the participating teachers were able to articulate specific teaching actions to facilitate learning. Some teachers were more specific than others and described in detail what would happen in the classroom to ensure that the cognitive demand of the task did not decline and that students were able to experience the related SMP in their problem solving. Figure 5 provides an example of the work from a teacher, Mrs. Behm, and her mapping of student outcomes to instruction with respect to a problem dealing with seats on a bus.

Mathematical Practice (Number and Title): 5 & 6  
Tools Being Thoughtful

Effect: Implications for learning opportunities If this is what I want to see my students doing...	Cause: Implications for teaching ...then this is what I need to plan for instruction.
<ul style="list-style-type: none"> <li>Use a 10 frame (which represents each bus) to see how many buses are needed.</li> <li>Use 10 frame to see how many empty seats are left</li> <li>Explain why a 10 Frame is a good tool to use</li> <li>Discussing with partner/group</li> </ul>	<ul style="list-style-type: none"> <li>Decide which tool to use to help solve the problem</li> <li>Use the tool for extension activities (Teacher got on bus... 2 students are sick...)</li> <li>Safe and supportive classroom environment where mistakes are learning opportunities.</li> <li>Model correct use/procedures</li> </ul>

Figure 5. Mrs. Behm’s considerations about how to implement ten-frames in her classroom

In this example, Mrs. Behm was specific in what she wanted students to be doing in saying, “Use a ten-frame (see Figure 6) to see how many empty seats are left.” She also provided a description of what she would have to do to make sure this happens, “Use the tool of extension activities...model correct use/procedures.” In this way, Mrs. Behm thought through what it would mean for students to solve the task she had modified, and she was able to decide what she needed to do as a teacher to support this learning for students. She realized that for students to be able to use appropriate tools strategically, she needed to provide opportunities for students to encounter these tools initially. As she thought about her teaching, she was mindful of how she would incorporate this practice to scaffold students’ progress toward selecting and using tools of their choice.

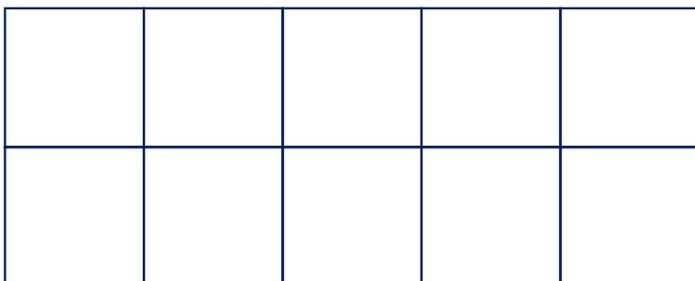


Figure 6. Ten-Frame

## In Your Classroom

We have described a process that engaged kindergarten, first-, and second-grade teachers in considering the cognitive demand of the tasks they use in their teaching. We have also detailed how they considered these tasks from the perspective of the SMPs with a focus on implications for both teaching and learning. While this process was completed over an extended period of time, it is a method you can immediately incorporate in your own classroom to ensure you are supporting students in developing their mathematical abilities. By purposefully engaging in this process a few times on your own or with colleagues, you will begin to deliberately plan for and use cognitively demanding tasks that support students’ development of the SMPs. To incorporate this process into your planning, first select a task. Next, modify the task to increase the cognitive demand. It is strongly recommended that you get peer feedback on this portion. After you then have a finalized task, select a mathematical practice as the focus of your lesson. Then, consider both the implications for learning and implications for teaching. It may take time to write down your ideas for this particular step, but the time is well spent to help you formulate and solidify ideas. Once you have considered both what you want to see students doing and your necessary actions to facilitate this type of learning, you are ready to implement the task. As you implement the task, be sure to maintain fidelity to your cause and effect notes to ensure that the task delivered is the task that was planned. This will help ensure that you maintain the level of cognitive

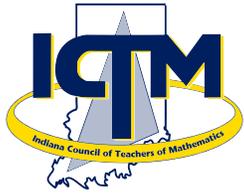
demand originally anticipated and that you incorporate the relevant SMP. A word of caution, however; the act of modifying a task does not ensure that it will increase its cognitive demand. It is recommended that you first check your ideas with a trusted colleague to ensure that your intended outcomes of the modification are accurate. With that said, do not hesitate to get started. If you reflect on your teaching while going through the process, you will learn to rethink the role of cognitively demanding tasks in the classroom.

## Cognitive Demand

Teaching with problems that require high levels of cognitive demand can be a difficult task. Likewise, maintaining high levels of demand during the implementation of tasks can also be challenging. Incorporating and integrating the SMPs can be additionally problematic if you do not have specific plans for how this may look in your classroom. As you prepare lessons, consider the cause and effect of the situation while realizing at the same time that kindergarten, first-, and second-grade tasks can be modified to appropriately increase cognitive demand. It is important to know that simple changes to the problems in the curricula you are using can improve opportunities for students’ learning. By providing tasks that are simultaneously higher in cognitive demand and that develop the SMPs, even very young students can “do” mathematics.

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