

# **Geometry / Integrated Math II**

## **2012**

**Sponsored by the Indiana Council of Teachers of Mathematics**

**Indiana State Mathematics Contest**

This test was prepared by faculty at **Indiana State University**

**ICTM Website**

**<http://www.indianamath.org/>**

Do not open this test booklet until you have been advised by the test proctor.

**Next year's math contest date: April 27, 2013**

- There is a circle inscribed in an equilateral triangle, and there is another equilateral triangle inscribed in the circle. The ratio of the area of the larger triangle to the smaller triangle is:  
 A) 81:16                  B) 9:4                  C) 2:1                  D) 4:1                  E) none of these
- Which of the following sets of 3 segments would make up the three sides of a scalene triangle?  
 I.      3 cm – 4 cm – 5 cm                                  II.      4 cm – 4 cm – 5 cm  
 III.    1 cm – 2 cm – 3 cm                                  IV.     12 cm – 13 cm – 7 cm  
  
 A) I and IV only                                  B) I, II, and III only                                  C) I, III, and IV only  
  
 D) all of these                                  E) none of these
- If the diameter of a sphere is doubled then the volume of the sphere is increased by a factor of :  
 A) 2                                  B) 4                                  C) 8                                  D) 16                                  E) none of these
- The distance between the two parallel lines which have equations  $y = \frac{3}{4}x + 2$  and  $y = \frac{3}{4}x - 1$  is:  
 A)  $\frac{\sqrt{2}}{5}$                                   B)  $\frac{2}{5}$                                   C) 1                                  D) 3                                  E) none of these
- The area of square  $ABCD$  below is 4. Diagonal  $AC$  is extended its own length to the point  $E$ , so that  $C$  is the midpoint of segment  $AE$ . The length of segment  $BE$  is:

- $2\sqrt{5}$
- 4
- $4\sqrt{2}$
- $2\sqrt{3}$
- none of these

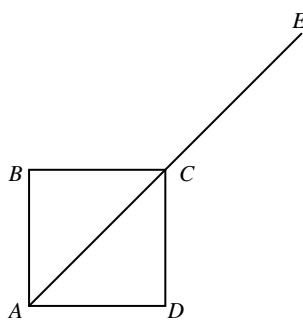


Figure for problem 5

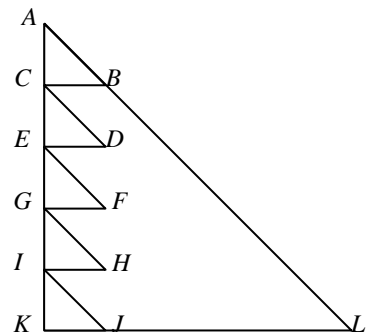


Figure for problem 6

- The triangles  $\Delta ABC, \Delta CDE, \Delta EFG, \Delta GHI, \Delta IJK$  in the figure above and to the right are congruent to each other and are similar to  $\Delta AKL$ . If the area of  $\Delta ABC$  is 4, then the area of  $\Delta AKL$  is:  
 A) 25                                  B) 60                                  C) 100                                  D) 120                                  E) none of these

7. If the sides of a triangle have length 55, 73, and 48, then the radius of the circle that circumscribes the triangle is:

A)  $\sqrt{73}$       B)  $\frac{103}{2}$       C)  $\frac{176}{3}$       D)  $\frac{73}{2}$       E) none of these

8. The area of the trapezoid in the figure to the right is:

A) 10  
 B) 6  
 C)  $2 + 2\sqrt{2}$   
 D)  $\frac{1}{2} + \frac{\sqrt{2}}{2}$   
 E) none of these

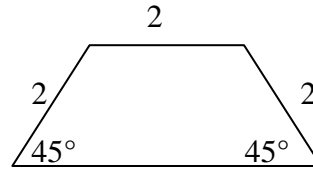


Figure for problem 8

9. A 6.5 m ladder is placed against a vertical wall of a building. The foot of the ladder is 1.6 m from the base of the building. If the top of the ladder slides down 0.7 m then the foot of the ladder will slide:

A) 0.7 m      B) 1.7 m      C) 3.3 m      D) 5.6 m      E) none of these

10. The sum of the number of vertices, edges, and faces of a regular dodecahedron is:

A) 40      B) 50      C) 60      D) 62      E) none of these

11. A rectangular prism is made of 2000 individual, unpainted, unit cubes and has dimensions  $10 \times 10 \times 20$ . If the prism is painted and then separated into the original unit cubes, the number of cubes with at least one side painted is:

A) 848      B) 912      C) 1002      D) 1152      E) none of these

12. The ratio of the length of the side of a regular pentagon to the length of its diagonal is:

A)  $1 : \sqrt{5}$       B)  $1 : 2\sqrt{3}$       C)  $\sqrt{3} : 2\sqrt{5}$       D)  $2 : \sqrt{5} + 1$       E) none of these

13. Suppose you have a map of Indiana with a scale of 1 in = 30 mi. You trace the map of the state, cut out your tracing, and draw this tracing onto a piece of  $8\frac{1}{2}'' \times 11''$  cardstock that weighs 20 grams. You then cut out the tracing of Indiana and weigh it. You find that it weighs 8.5 grams. The approximate area of Indiana in square miles is:

A) 1,200      B) 20,000      C) 36,000      D) 60,000      E) 144,000

14. The figure to the right is a net of a right triangular prism. The lateral surface area of the prism in square units is:

A) 48  
 B) 60  
 C) 72  
 D) 84  
 E) none of these

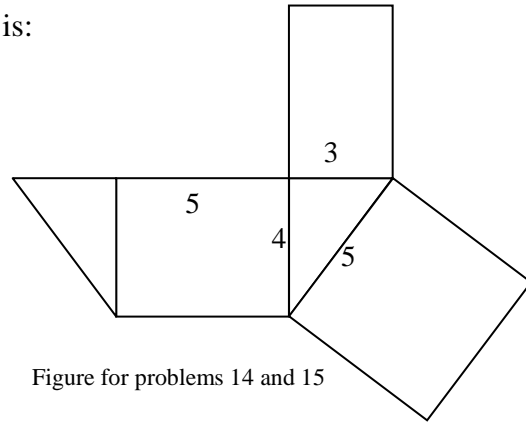


Figure for problems 14 and 15

15. The figure to the right and above is a net of a right triangular prism. The volume of the prism in cubic units is:
- A) 30                      B) 60                      C) 90                      D) 120                      E) none of these
16. The ratio of the length of the shorter diagonal of a regular hexagon to the length of its longer diagonal is:
- A)  $\sqrt{3} : 1$               B)  $\sqrt{3} : 2$               C)  $\sqrt{3} : 3$               D)  $\sqrt{3} : 4$               E) none of these
17. Triangle  $RST$  has vertices given by  $R(0,0)$ ,  $S(a, 0)$ , and  $T(0, b)$ . In terms of  $a$ ,  $b$ , and  $c$ , the coordinates of the centroid of the triangle are:
- A)  $\left(\frac{a}{3}, \frac{b}{3}\right)$               B)  $\left(\frac{a}{2}, \frac{b}{2}\right)$               C)  $\left(\frac{b}{3}, \frac{a}{3}\right)$               D)  $\left(\frac{b}{2}, \frac{a}{2}\right)$               E) none of these
18. A cone and a cylinder share a base, have the same volume, and the height of the cylinder is 2 cm. The height of the cone is:
- A) 12 cm                      B) 8 cm                      C) 6 cm                      D) 4 cm                      E) none of these
19. In right triangle  $ABC$ ,  $\tan A = \frac{3}{4}$ . The sum,  $\cos A + \sec A$  is equal to:
- A) 1                              B)  $\frac{34}{15}$                       C)  $\frac{41}{20}$                       D)  $\frac{37}{15}$                       E) none of these

20. Which of the following statements is always true?
- I. The diagonals of a rhombus are congruent.
  - II. The diagonals of a rhombus are perpendicular.
  - III. The diagonals of a rhombus bisect each other.
  - IV. The diagonals of a rhombus bisect the vertex angles of the rhombus.
- A) All of the statements are true.
  - B) All but IV are true.
  - C) All but I are true.
  - D) Only II and III are true.
  - E) None of the statements are true.
21. In a rectangle, the length of one of the sides is 77 cm. The sum of the lengths of a diagonal and another side is 121 cm. The area of this rectangle in square centimeters is:
- A) 6545
  - B) 5929
  - C) 2772
  - D) 2464
  - E) none of these
22. The equation of a circle with center  $(-3, 1)$  and radius 25 is:
- A)  $(x + 3)^2 + (x - 1)^2 = 5$
  - B)  $(x + 3)^2 + (x - 1)^2 = 25$
  - C)  $(x + 3)^2 - (x - 1)^2 = 625$
  - D)  $(x - 3)^2 + (x + 1)^2 = 625$
  - E) none of these

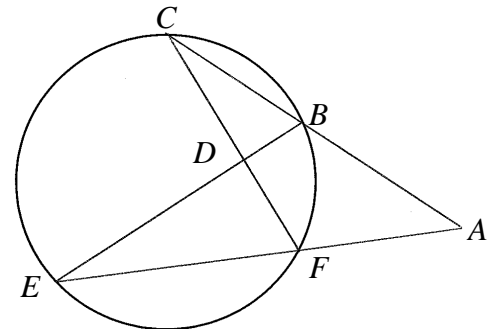


Figure for problem 23

23. In the figure above on the right,  $m\angle EAC = 40^\circ$  and  $m\angle EDC = 90^\circ$ . The sum of the measures of minor arc  $EC$  and minor arc  $FB$  is
- A)  $50^\circ$
  - B)  $100^\circ$
  - C)  $130^\circ$
  - D)  $180^\circ$
  - E) none of these
24. The locus of points whose ratio of the distance from a fixed line to the distance from a fixed point is less than 1 is
- A) a parabola
  - B) a hyperbola
  - C) an ellipse
  - D) a circle
  - E) none of these

25. In the figure below and to the right, quadrilateral  $ABCD$  is inscribed in a circle. Which of the statements are always true?

- I.  $m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$
- II.  $m\angle A + m\angle B = 180^\circ$
- III.  $m\angle A + m\angle C = 180^\circ$

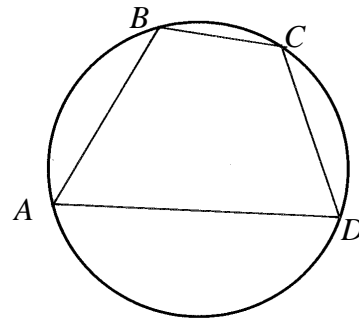


Figure for problem 25

- A) I only
- B) I and II only
- C) I and III only
- D) II and III only
- E) none of these

26. The figure below on the left is a hexafoil that was created from 7 congruent triangles. The center of the middle circle is a point on each of the outer circles. If the radius of each of the circles is 3 cm, the perimeter, in cm, of the shaded region is:

- A)  $12\pi$
- B)  $9\pi$
- C)  $6\pi$
- D)  $3\pi$
- E) none of these

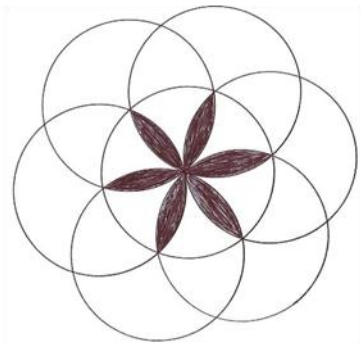


Figure for problem 26

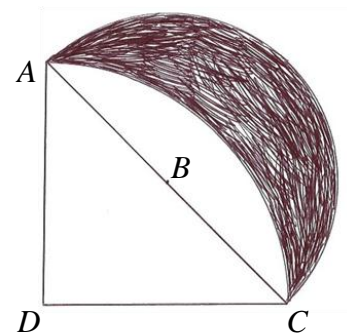


Figure for problem 27

27. In the figure above and to the right, points  $B$  and  $D$  are centers of circular arcs,  $\angle ADC$  is a right angle, and  $AC = 12$  cm. The shaded region is a lune. The area of the lune in square cm is:

- A)  $40\pi$
- B)  $18\pi$
- C)  $12\pi$
- D) 36
- E) none of these

28. The surface area of a large cube is 9600 square cm. This cube is cut into a number of identical smaller cubes. Each smaller cube has a volume of 512 cubic cm. The number of smaller cubes is:

- A) 5
- B) 19
- C) 125
- D) 150
- E) none of these

29. In the figure below and to the left, two perpendicular line segments divide a large rectangle into 4 smaller rectangles. The areas of 3 of these 4 small rectangles are shown. The area of the fourth small rectangle is:

A) 10                      B) 12                      C) 15                      D) 18                      E) none of these

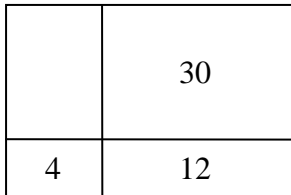


Figure for problem 29

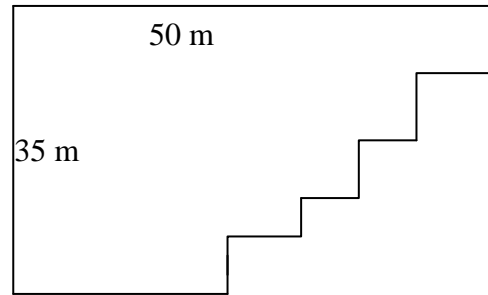


Figure for problem 30

30. The dodecagon in the figure above and to the right has interior angles measuring  $90^\circ$  and  $270^\circ$ . The perimeter of the dodecagon, in meters, is:

A) 85                      B) 170                      C) 215                      D) 875                      E) none of these

31. If the lengths of two sides of a right triangle are 5 and 12 the least possible length of the third side is:

A)  $\sqrt{60}$                       B)  $\sqrt{119}$                       C) 13                      D) 16                      E) none of these

32. In a circle of radius  $r$ , two parallel chords of length  $r$  form opposite sides of a rectangle. The area of the rectangle is:

A)  $r^2\sqrt{2}$                       B)  $r^2\sqrt{3}$                       C)  $r^2\sqrt{5}$                       D)  $3r^2$                       E) none of these

33. The volume of a cone, in cubic centimeters, made from a circular sector of radius 3 cm and central angle  $40^\circ$  is:

A)  $\frac{4\sqrt{5}\pi}{81}$                       B)  $\frac{10\pi}{9}$                       C)  $\frac{4\sqrt{5}\pi}{243}$                       D)  $9\pi$                       E) none of these

34. The number of positive integers  $n$  that have the property that the measures, in degrees, of the interior angles of a regular  $n$ -gon are integers is:

- A) 8                      B) 20                      C) 22                      D) 35                      E) none of these

35. In the figure below and on the left,  $\overline{AB} \parallel \overline{FC} \parallel \overline{ED}$ . The length of segment  $AB$  is 10 and the length of segment  $ED$  is 7. The length of segment  $FC$  is:

- A)  $4\frac{2}{17}$                       B)  $4\frac{2}{3}$                       C)  $3\frac{5}{14}$                       D)  $2\frac{2}{15}$                       E) none of these

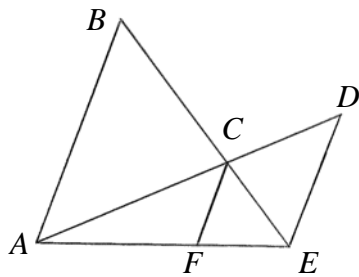


Figure for problem 35

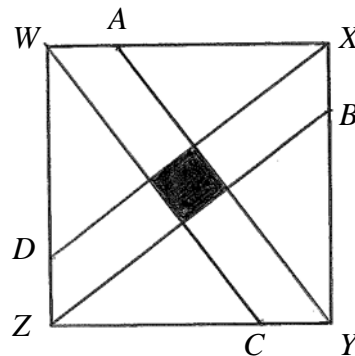


Figure for problem 36

36. The area of the square  $WXYZ$  in the figure above and on the right is equal to 1. The points  $A$ ,  $B$ ,  $C$ , and  $D$  divide the sides of the square in a 3:1 ratio. The area of the shaded region is:

- A)  $\frac{1}{22}$                       B)  $\frac{1}{25}$                       C)  $\frac{1}{36}$                       D)  $\frac{2}{49}$                       E) none of these