Geometry / Integrated Math II

2011

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

ICTM Website

http://www.indianamath.org/

Do not open this test booklet until you have been advised by the test proctor.

Next year's math contest date: April 28, 2012

1. Two angles are supplementary. One angle is 40° more than four times the other. Find the difference of the measures of the two angles.

A) 10° B) 100° C) 124° D) 106° E) none of these

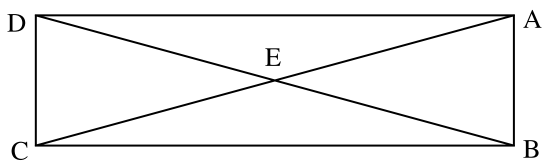
2. Which of the following are ***never*** true?

I. For points A, M and B, if $AM = MB$, then A, M, and B are collinear.
II. If two angles are congruent, then they are right angles.
III. The bisectors of vertical angles are opposite rays.
IV. The supplement of an obtuse angle is another obtuse angle.
V. Complementary angles are congruent.

A) I only B) I, III, and IV C) III, IV, and V

D) IV only E) none of these

3. Quadrilateral ABCD is a rectangle. Which of the following statements are true?



A) Area of Δ CDE = Area of Δ ABE

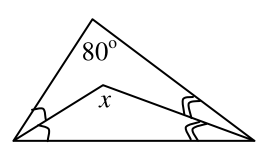
B) Area of Δ CDE = Area of Δ DEA

C) Area of Δ CDE = Area of Δ CEB

D) All of the above are true

E) none of these

4. Find the value for x in degrees in the figure to the right (not drawn to scale) that has congruent angles marked.



A) 80°

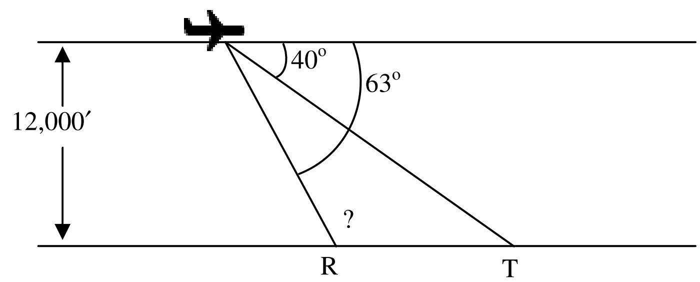
B) 130°

C) 160°

D) 125°

E) none of these

5. An airplane has leveled off and is flying horizontally at an altitude of 12,000 feet. The pilot can see each of two small towns at R and T in front of the plane. With angle measures as indicated in the figure below (not drawn to scale), find the measure of $\angle R$.

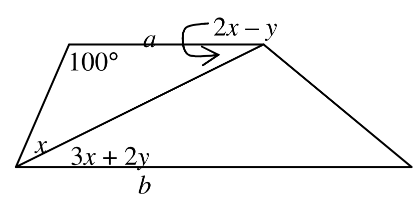


A) 117° B) 63° C) 103° D) 77° E) none of these

6. Suppose that (a) distinct planes M and N intersect, (b) point A lies in both planes M and N, and (c) point B lies in both planes M and N. What can you conclude about $\overleftrightarrow{AB}$?

A) $\overleftrightarrow{AB}$ is perpendicular to plane M only
B) $\overleftrightarrow{AB}$ is the intersection of planes M and N
C) $\overleftrightarrow{AB}$ is perpendicular to plane N only
D) $\overleftrightarrow{AB}$ is perpendicular to both plane M and plane N
E) none of these

7. Given the figure to the right where a and b are lines and $a \parallel b$, find the values of x and y.



A) $x = 30$, $y = -10$
B) $x = 24$, $y = 8$
C) $x = 30$, $y = 10$
D) $x = 24$, $y = -8$
E) none of these

8. Which of the following are ***always*** true?

I. An isosceles triangle is a right triangle.
II. A right triangle has two complementary angles.
III. An equilateral triangle is a right triangle.
IV. A scalene triangle is an isosceles triangle.
V. A right triangle has two congruent angles.

A) IV only B) II and V C) I, III, and V D) II only E) none of these

9. Which of the following are ***always*** true?

I. A square is a rectangle.
II. If two of the angles of a trapezoid are congruent, then the trapezoid is isosceles.
III. The diagonals of a rhombus are perpendicular bisectors of each other.
IV. Two consecutive angles of a parallelogram are supplementary.
V. The four sides of a kite are congruent.

A) I only B) I, II, III, and IV C) I, III, and IV
D) IV only E) none of these

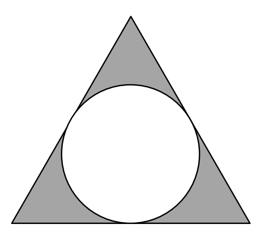
10. Which of the following statements are ***never*** true for circles with positive radii?

I. In a circle, congruent chords are equidistant from the center.
II. If a central angle and an inscribed angle of a circle intercept the same arc, then they are congruent.
III. If a parallelogram is inscribed in a circle, then each of its diagonals must be a diameter.
IV. If two chords of a circle are not congruent, then the shorter chord is nearer the center of the circle.
V. Concentric circles have exactly one point in common.

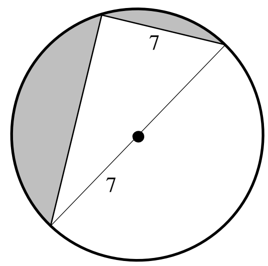
A) II and IV B) II and V C) II, III, and IV
D) II, IV, and V E) none of these

11. Find the area of the shaded part of the region of the figure below in square units. The figure is a circle inscribed in an equilateral triangle whose sides have length 10 units.

A) $25\sqrt{3} - \frac{50}{9}\pi$
B) $25\sqrt{3} - \frac{25}{9}\pi$
C) $25\sqrt{3} - \frac{25}{9}\pi$
D) $25\sqrt{3} - \frac{25}{3}\pi$
E) none of these



12. Find the area of the shaded region of the figure below in square units.



A) $49\pi - 49\sqrt{3}$

B) $49\pi - \frac{49}{2}\sqrt{3}$

C) $\frac{49}{2}\pi - \frac{49}{2}\sqrt{3}$

D) $49\sqrt{3} - 49\pi$

E) none of these

13. How many vertices does a pentagonal prism have?

A) 5 B) 6 C) 7 D) 8 E) none of these

14. How many faces does a right octagonal pyramid have?

A) 6 B) 7 C) 8 D) 9 E) none of these

15. How many edges does an oblique hexagonal prism have?

A) 18 B) 20 C) 22 D) 24 E) none of these

16. Find the length of the radius of a right circular cone made from a circular sector with a radius of 12 and a central angle with measure $150°$ whose radii are joined together to make the lateral part of the cone.

A) 4 B) 5 C) 6 D) 7 E) none of these

17. When a circular region is revolved about a line on the circle's exterior, what is the name of the resulting shape?

A) sphere B) torus C) parabola D) frustrum E) none of these

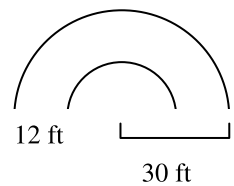
18. Describe the shape of the faces of a regular octahedron.

A) triangles B) squares C) pentagons D) octagons E) none of these

19. A lawn roller in the shape of a right circular cylinder has a diameter of 18 in. and a length of 4 ft. Find the area rolled during one complete revolution of the roller. Give your answer to the nearest square foot.

A) 6 B) 19 C) 72 D) 226 E) none of these

20. Sue and Dave's semicircular driveway below (not drawn to scale) is to have flowers planted on both of the curved sides. If individual flowers are to be planted 1 foot from the edge of the driveway at intervals of approximately 1 foot, how many flowers are needed (rounded to the nearest 10)?



A) 150 B) 200 C) 250 D) 300 E) none of these

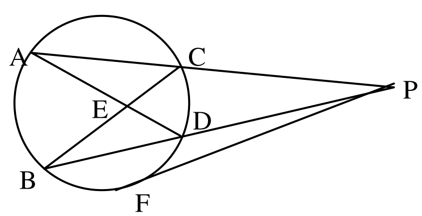


Figure for 21, 22, and 23

21. In the figure above (not drawn to scale), the measure of minor arc *AB* is 80° and the measure of angle *AEB* is 75°. What is the measure of minor arc *CD*?

A) 65° B) 70° C) 75° D) 80° E) none of these

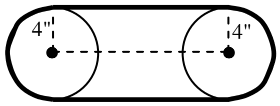
22. In the figure above (not drawn to scale), if $AE = 8$, $ED = 5$, $CE = 10$, what is BE?

A) 4 B) 2.5 C) 16 D) 8 E) none of these

23. In the above figure (not drawn to scale), if $FC = 6$ and $ED = 9$, what is CD?

A) 3 B) 2 C) 12 D) 18 E) none of these

24. If two circular gears (pictured in the figure below that is not drawn to scale), each of radius 4 in, are used in a chain drive system with a chain of length 54 in, what is the distance in inches (to the nearest hundredth) between the centers of the gears?



A) 20.22 B) 16.32 C) 14.43 D) 18.12 E) none of these

25. Find the altitude (in meters) of a right circular cone in which the diameter of the base measures 9.6 m and the slant height is 5.2 m.

A) 6 B) 5 C) 4 D) 3 E) none of these

26. The total surface area of a regular hexahedron is 105.84 m^2. Find the length of each edge in meters (to the nearest tenth).

A) 10.2 B) 8.0 C) 6.4 D) 4.2 E) none of these

27. A sphere is inscribed within a right circular cylinder whose altitude and diameter have equal measures. Find the ratio of the surface area of the cylinder to that of the sphere.

A) 2:1 B) 3:1 C) 3:2 D) 2:3 E) none of these

28. If (2,3), (5,-2), and (7,2) are three vertices (not necessarily consecutive) of a parallelogram, find the possible locations of the fourth vertex.

A) (5,7); (0,-2); (9,-3) B) (3,6); (1,-2); (9,-4) C) (4,7); (0,-1); (10,-3)

D) (3,5); (1,-3); (8,-4) E) none of these

29. There are two points on the *y* axis that are located a distance of 6 units from the point (3,1). Determine the coordinates of each point.

A) $(0, 1+3\sqrt{3})$ and $(0, 1-3\sqrt{3})$

B) $(0, 2+\sqrt{3})$ and $(0, 2-\sqrt{3})$

C) $(0, 3\sqrt{3}+1)$ and $(0, 3\sqrt{3}-1)$

D) $\left(0, \quad 1+\frac{\sqrt{3}}{2}\right)$ and $\left(0, \quad 1-\frac{\sqrt{3}}{2}\right)$

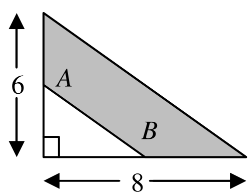
E) none of these

30. For triangle *PNQ*, the vertices are *P*(0, 0), *N*(*a*, 0), and *Q*(*b*, *c*). In terms of *a*, *b*, and *c*, find the coordinates of the orthocenter of triangle *PNQ*.

A) $\left(\frac{a-b}{c}, \quad a\right)$ B) $\left(b, \quad \frac{ab-b^2}{c}\right)$ C) $\left(\frac{b^2-4ac}{2a}, \quad b\right)$

D) $\left(\frac{b-a}{c}, \quad a\right)$ E) none of these

31. The shaded region in the figure below (not drawn to scale) is that of a trapezoid. Determine the height of the trapezoid if *A* and *B* are midpoints of their respective sides.



A) 4.2 units B) 3.1 units C) 2.4 units D) 1.3 units E) none of these

32. The locus of points that are equidistant from a fixed line and a point not on that line is called:

A) a parabola B) a hyperbola C) an ellipse D) a circle E) none of these