Geometry/Integrated Math II

2010

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

ICTM Website

http://www.indiana math.org/

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Next year's math contest date: April 23, 2011

1) Which of the following sets of 3 segments would make up the three sides of a right triangle?

I.	3 cm – 4 cm – 5 cm	II.	4 cm – 4 cm – 5 cm
III.	13 cm – 12 cm – 5cm	IV.	24 cm – 7 cm – 25 cm

a. I only
b. I and II only
c. I, II, and III only
d. I and IV only
e. I, III, and IV only

2) What is the product of the coordinates of the centroid of the triangle whose vertices have coordinates $(0,0)$, $(8,0)$, and $(4,-4)$?

a. $-\frac{8}{3}$ b. $-\frac{16}{3}$ c. $-\frac{32}{3}$ d. -4 e. none of these

3) The ratio of the length of the altitude of an equilateral triangle to the length of its side is

a. $\sqrt{3}:2$ b. $2:\sqrt{3}$ c. $1:2$ d. $2:1$ e. none of these

4) The following figure (not necessarily to scale) consists of two squares with side length of 12 and one square with a side length of 8. What is the area of the shaded part of the figure in square units?

a. 144

b. 72

c. 48

d. 36

e. none of these

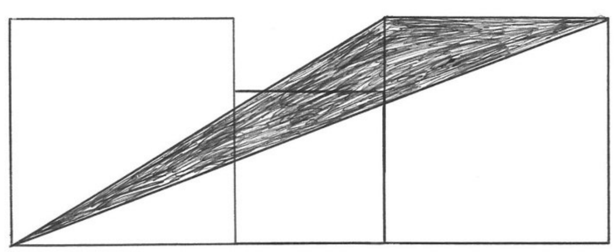

figure for problem 4

5) Consider the following diagram (not necessarily to scale). $AB = 8$, $AC = 7$, $BD = 4$, and $\angle BAD \cong \angle CAD$. What is the length of $\overline{CD}$?

a. $3\frac{1}{2}$

b. 4

c. $4\frac{1}{3}$

d. $4\frac{1}{2}$

e. none of these

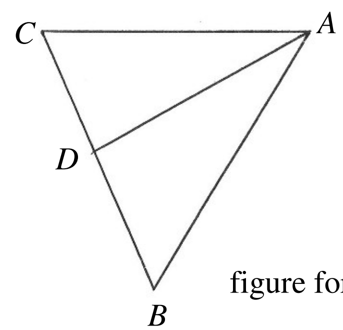

figure for problem 5

6) Consider the following combinations of sides and angles of a triangle. Which of these can be used to prove congruence of two triangles?

I. Side – Angle – Side II. Side – Side – Side III. Angle – Angle – Angle
IV. Side – Side – Angle V. Angle – Angle – Side VI. Angle – Side – Angle

a. all of them

b. all but III

c. I, II and VI only

d. All but III and IV

e. none of these

7) The following hexafoil was created from 7 congruent circles. The center of the middle circle is a point on each of the outer circles. If the radius of each of the circles is 2 units, what is the area of the shaded region in square units?

a. $\frac{2}{3}\pi$

b. $\frac{2}{3}\pi - \sqrt{3}$

c. $4\pi - 6\sqrt{3}$

d. $8\pi - 12\sqrt{3}$

e. none of these

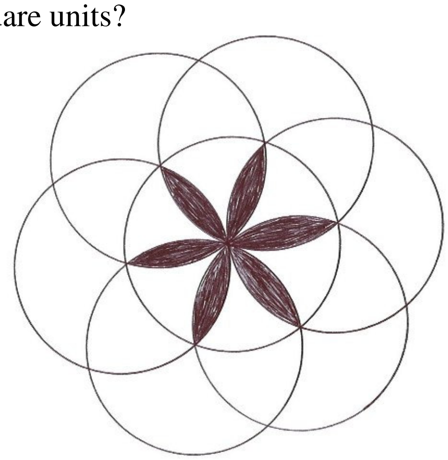
figure for problem 7

8) If the diagonals of a quadrilateral are perpendicular bisectors of each other, the figure would always be included under the general classification:

a. rhombus b. rectangle c. square d. concave quadrilateral e. none of these

9) One of the legs of a right triangle has length $\sqrt{19}$ cm. The sum of the lengths of the hypotenuse and the other leg is 19 cm. What is the area of this triangle in square centimeters?

a. $4\sqrt{19}$ b. $5\sqrt{19}$ c. $6\sqrt{19}$ d. $7\sqrt{19}$ e. none of these

10) The sum of the numbers of vertices, edges, and faces of an octagonal pyramid is

a. 26 b. 34 c. 50 d. 65 e. none of these

11) What is the volume, in cubic centimeters, of a cone without a base that was made from a semi-circle of radius 12 cm?

a. 72π b. $72\sqrt{2}\,\pi$ c. $72\sqrt{3}\,\pi$ d. 96π e. none of these

12) A circle has a chord of length 10 cm. This chord is the perpendicular bisector of a radius of that circle. What is the area of the circle in square centimeters?

a. 25π b. $\frac{100}{3}\pi$ c. 50π d. 100π e. none of these

13) One thousand unit cubes are fastened together to form a large cube. This large cube is painted and then separated into the original cubes. The number of cubes that have zero faces painted is:

a. 0 b. 72 c. 384 d. 512 e. none of these

14) The following figure is that of a regular octagon adjacent to a regular decagon. The side lengths of the decagon and the octagon are the same. The measure of $\angle ABC$ is:

a. $49.5°$

b. $50°$

c. $81°$

d. $99°$

e. none of these

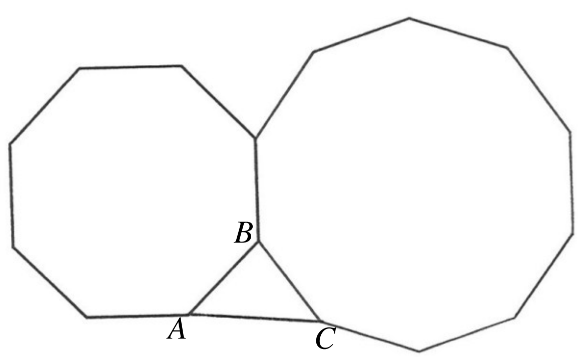

figure for problem 14

15) The equation of a circle with center $(4, -5)$ and radius 16 is

a. $(x-4)^2 + (y+5)^2 = 256$

b. $(x-4)^2 + (y+5)^2 = 32$

c. $(x+4)^2 + (y-5)^2 = 16$

d. $(x-4)^2 + (y+5)^2 = 4$

e. none of these

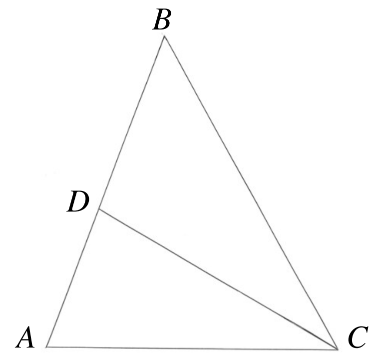

figure for problem 16

16) In the figure above and to the right (not necessarily drawn to scale), $\overline{CD}$ bisects $\angle BCA$, $m\angle BAC = 70°$, $m\angle ABC = 50°$. If $x = m\angle DCA$ and $y = m\angle BDC$, then the sum $x + y$ is

a. 150° b. 140° c. 130° d. 120° e. none of these

17) In the figure to the right, $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and $\overline{AE} \cong \overline{EF} \cong \overline{FG}$. If $FC = 8$, $x = EB$, and $y = GD$, then the product xy is equal to

a. 36
b. 48
c. 60
d. 72
e. none of these

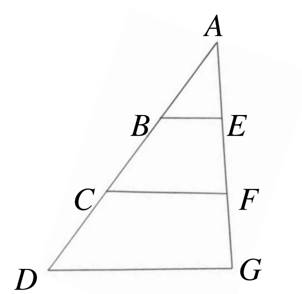

figure for problem 17

18) In the figure to the right (not necessarily to scale), ΔABC is inscribed in the circle. $\overline{AB} \cong \overline{AC}$, $\overline{DE}$ is tangent to the circle at point *C*, and the measure of arc *AC* is 140°. The measure of $\angle BCE$ is:

a. 20°
b. 40°
c. 70°
d. 80°
e. none of these

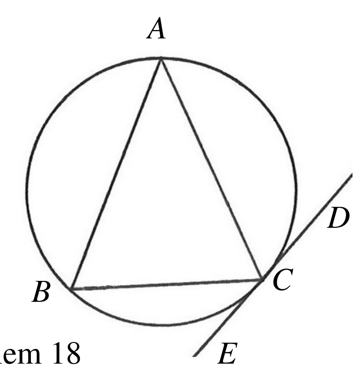

figure for problem 18

19) If the radius of a circle is increased 200%, then the area is increased:

a. 200% b. 400% c. 800% d. 40,000% e. none of these

20) In the figure below and to the left (not necessarily drawn to scale), the circle with center O is inscribed in quadrilateral $ABCD$. Points P, Q, R, and S are the points of tangency. If $BQ = 27$, $BC = 38$, $OP = 10$, and $\angle ADC$ is a right angle, then DC is equal to:

a. 10 b. 11 c. 20 d. 21 e. none of these

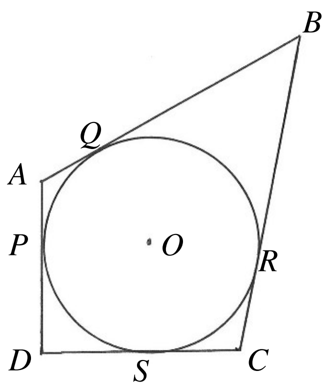

figure for problem 20

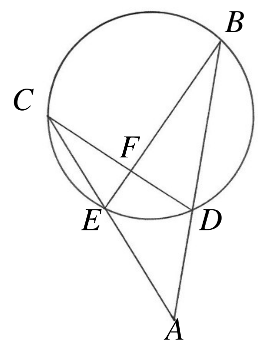

figure for problem 21

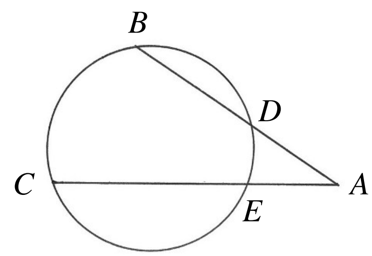

figure for problem 22

21) In the figure above and center (not necessarily drawn to scale), $m\angle BAC = 50°$ and $m\angle BFC = 75°$. The difference between the measures of minor arc BC and minor arc DE is:

a. 100° b. 125° c. 25° d. 62 ½° e. none of these

22) In the figure above and to the right (not necessarily drawn to scale), $AB = 14$, $AC = 18$, and $AE = 5$. AD is equal to:

a. $\frac{45}{7}$ b. $\frac{35}{9}$ c. $\frac{32}{5}$ d. 6 e. none of these

23) A point is 12 centimeters from the center of a circle whose radius is 13 centimeters. The sum of the lengths of the longest and shortest chord that can be drawn through this point is:

a. 18 cm b. 26 cm c. 36 cm d. 52 cm e. none of these

24) The radius of the circumcircle of a triangle whose sides have length 29, 21, 20 is

a. 14 ½ b. 10 ½ c. 10 d. 23 e. none of these

25) The distance between the two parallel lines which have equations $y = 4x + 10$ and $y = 4x - 3$ is

a. $\frac{13}{\sqrt{17}}$ b. 13 c. $\frac{13}{2}$ d. $\frac{\sqrt{13}}{17}$ e. none of these

26) In a rhombus, one diagonal is 3 times the length of the other diagonal. The length of a side of the rhombus in terms of A, where A is the area of the rhombus is:

a. $\frac{\sqrt{3A}}{3}$ b. $\frac{\sqrt{9A}}{3}$ c. $\frac{\sqrt{15A}}{3}$ d. $\frac{\sqrt{21A}}{3}$ e. none of these

27) In the figure below on the left (not necessarily drawn to scale, points B and D are centers of circular arcs, $\angle ADC$ is a right angle, and $AC = 40$ cm. The shaded region is a lune. The perimeter of the lune is:

a. 10π b. 20π c. $(20-10\sqrt{2})\pi$ d. $(20+10\sqrt{2})\pi$ e. none of these

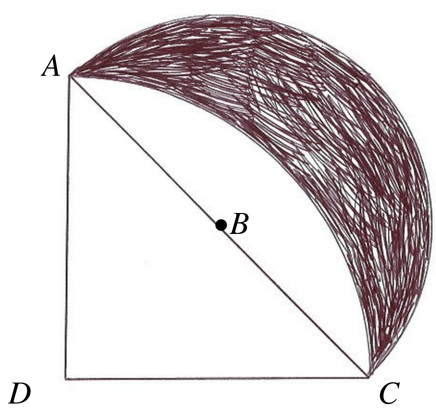

figure for problem 27

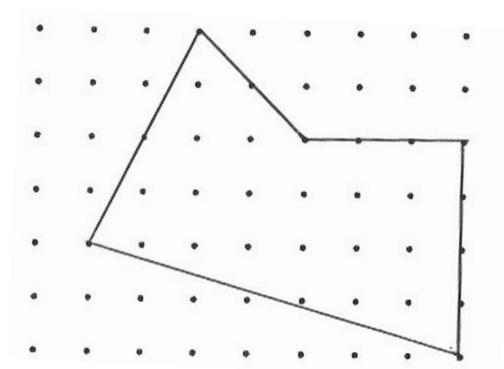
figure for problem 28

28) In the figure above on the right, the dots are one unit apart in both a horizontal and vertical direction. The area, in square units, of the figure is:

a. 18 b. 23 c. 34 d. 42 e. none of these

29) In right triangle ABC, $\tan A = \frac{12}{5}$. The sum, $\sin A + \cos A$ is equal to

a. 1 b. $\frac{17}{13}$ c. $\frac{60}{169}$ d. $\frac{60}{17}$ e. none of these

30) In the figure below, $\overline{AB} \parallel \overline{EF}$. $m\angle ABC + m\angle BCD + m\angle CDE + m\angle DEF$ is equal to:

a. 180° b. 360° c. 540° d. 720° e. none of these

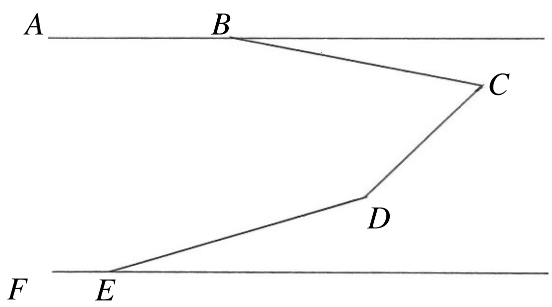

figure for problem 30

31) The ratio of the length of the longer diagonal of a regular hexagon to the length of its side is:

a. $2:1$ b. $1:2$ c. $\sqrt{3}:1$ d. $1:\sqrt{3}$ e. none of these

32) Quadrilateral *ABCD*, below has been dissected into squares. The area of the square with the light shading is 64 square units. The area of the square with the dark shading is 81 square units. The area, in square units, of quadrilateral *ABCD* is:

a. 1,000 b. 1,003 c. 1024 d. 1056 e. none of these

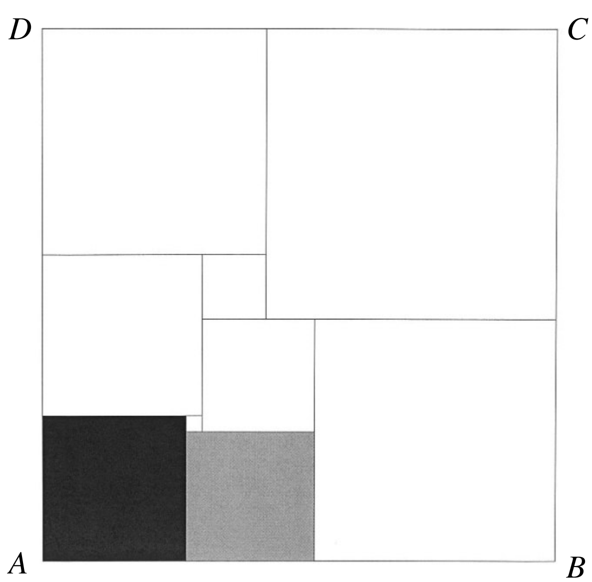

figure for problem 32