Algebra II / Integrated Math III

2009

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

ICTM Website

http://www.indianamath.org/

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Next year's math contest date: April 24, 2010

1) Solve for x: $(x+1)(x+2)(x+3) = 2(x+1)(x+2)(x+4)$.

A) {-1, -2, -5}
B) {-5}
C) {-1, -2}
D) {-1, -2, -3, -4}
E) None of these

2) What is the maximum number of intersection points of the graph of a second degree polynomial function (a parabola) and the graph of a fourth degree polynomial?

A) 2 B) 3 C) 4 D) 6 E) infinitely many

3) For the function $f(x) = x^4 + ax^3 + bx^2 + cx + 2$ where -1 is the triple root of $f(x)$, what is a?

A) -2 B) -3 C) 2 D) 5 E) 6

4) If $f(x) = \dfrac{x^5 + x^2}{x^3 + 1}$, then $f(i)$, where $i = \sqrt{-1}$, is equal to:

A) -2 B) -1 C) 0 D) 1 E) None of these

5) How many subsets of {1, 2, 3, 4, 5, 6, 7} contain at least one even number?

A) 127 B) 15 C) 64 D) 35 E) 112

6) If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are perpendicular, the value of a is

A) $-\frac{2}{3}$ B) $-\frac{3}{2}$ C) 6 D) -6 E) None of these

7) The sum of the solutions of $|3x - 8| = |2x - 7|$ is

A) 1 B) 3 C) 4 D) 7 E) None of these

8) The polynomial $P(x)$ is of degree six and $P(-1)=3$ and $P(-1)=8$. What is the remainder when $P(x)$ is divided by $(x-1)$?

A) 3 B) -3 C) 8 D) -8 E) None of these

9) Written with no negative exponents, $\left(a^{-1}+b^{-1}\right)^{-1}$ is equal to

A) $a+b$ B) $\frac{a+b}{a\cdot b}$ C) $\frac{a\cdot b}{a+b}$ D) $\frac{a^2\cdot b^2}{a^2+b^2}$ E) $\frac{a^2+b^2}{a^2\cdot b^2}$

10) For what positive value of r will the graph of $y=2x$ and $2x^2+y^2=r^2$ intersect at (2, 4)?

A) $2\sqrt{3}$ B) 4 C) $3\sqrt{2}$ D) $2\sqrt{6}$ E) $4\sqrt{3}$

11) If $\log_3 x+\log_3(x^3)=8$ then $x=$

A) $\sqrt{3}$ B) 3 C) 9 D) 81 E) None of these

12) If $a+b>0$ and $c+d>0$, which of the following must be true?

A) $a+b+c>0$

B) $ac+bd>0$

C) $a+b>c+d$

D) $a^2+b^2>c^2+d^2$

E) $a^2+b^2>0$

13) Two lines have slopes whose sum is 5 and both go through the point (1,1). If one line goes through $(5, y_1)$ and the other line goes through $(5, y_2)$ then $y_1 + y_2$ is equal to

A) 22 B) 13 C) 6 D) 25 E) 50

14) Find the solution set for $\sqrt{2x-3} = -\sqrt{3x-2}$.

A) {1} B) {-1} C) {1, -1} D) { } E) None of these

15) If $\dfrac{x^3+a}{x-1} = rx^2 + sx + t + \dfrac{u}{x-1}$, then $r+s+t+u$ is equal to

A) 1 B) 3 C) 4 D) $a+3$ E) $a+4$

16) Let $f(x) = x^x$. What is $f(a+b)$?

A) $(a+b)^a (a+b)^b$

B) $a^a + b^b$

C) $a^a + b$

D) $(a+b)^x$

E) $a^a \cdot b^b$

17) The number of solutions, x, such that $|x| + |x-1| = 1$ is:

A) 0 B) 1 C) 2 D) 4 E) Infinite

18) The radius of the circle $x^2 - 4x + y^2 + 2y = 4$ is:

A) 1 B) 2 C) 3 D) 4 E) 5

19) The polynomial $p(x) = x^3 + 3x^2 - 6x - 8$ has

A) 3 integer zeros

B) 1 integer and 2 non-integer rational zeros

C) 1 integer and 2 irrational zeros

D) 3 irrational zeros

E) 1 integer and 2 complex (non-real) zeros

20) $\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)\cdot\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)$ is equal to

A) $8+2\sqrt{6}$

B) $2\sqrt{6}$

C) $2\sqrt{10}+2\sqrt{15}+2\sqrt{6}$

D) $-2\sqrt{6}$

E) $2\sqrt{30}$

21) Let $f(x) = \begin{cases} x, & x<1 \\ x+1, & 1 \le x < 3 \\ x+3, & x \ge 3 \end{cases}$. Then $f(0)+f(2)+f(4)$ is equal to

A) 0 B) 9 C) 10 D) 21 E) None of these

22) Solve the equation for x: $y = 2e^{2x} - 1$.

A) $\frac{1}{4}\ln\frac{y}{2}$ B) $\frac{1}{2}\ln\frac{y}{2}$ C) $\ln\frac{y}{2}$ D) $\frac{1}{2}\ln\frac{y+1}{2}$ E) $\ln\frac{y+1}{2}$

23) Two positive integers have a sum of 24, and their squares have a difference of 192. What is their positive difference?

A) 6 B) 8 C) 10 D) 12 E) 14

24) What is the sum of the solutions of $\sqrt[3]{x^2}+\sqrt[3]{x}=6$?

A) -19 B) -18 C) 6 D) 8 E) 18

25) If $f(x^2)=4\cdot x\cdot f(x+2)+3$, what is the value of $f(4)$?

A) -1 B) $-\frac{3}{7}$ C) 2 D) $\frac{7}{3}$ E) $\frac{22}{7}$

26) If $(1+r)^t=2$, then t is equal to

A) $\frac{2}{\sqrt{1+r}}$ B) $\frac{2}{1+r}$ C) $\sqrt{\frac{2}{1+r}}$ D) $\frac{2}{\ln(1+r)}$ E) $\frac{\ln(2)}{\ln(1+r)}$

27) How many times will the graph of $|x|+|y|=2$ intersect the graph of $x^2-y=2$?

A) 1 B) 2 C) 3 D) 4 E) 5

28) Given that $x^2+y^2=10$, $\sqrt[4]{xy}+\sqrt{xy}+27=29$, $x>0$, and $y>0$. What is $x+y$?

A) $2\sqrt{3}$

B) $3\sqrt{2}$

C) $\frac{1}{2}$

D) $\frac{\sqrt{3}}{2}$

E) 8

29) If $x+y=i$ and $x\cdot y=i$, where $i=\sqrt{-1}$, determine the value of x^3+y^3.

A) $3+i$ B) $i+1$ C) 6 D) 2 E) $3-i$

30) If $\log\left(x^2-1\right)-\log\left(x-1\right)=2$, solve for *x*.

A) 80 B) 99 C) $2^{10}+1$ D) 101 E) None of these

31) Find all *x* so that $\dfrac{x-3}{x+5}<0$.

A) $(-\infty,-5)$ B) $(-\infty,3)$ C) $(3,\infty)$ D) $(-3,5)$ E) $(-5,3)$

32) If $f(x)=1-4x$ and $f^{-1}(x)$ is the inverse function of $f(x)$, then $f(-3)\cdot f^{-1}(-3)$ is equal to

A) 1 B) 3 C) 4 D) 10 E) 13

33) What is the sum of the following infinite geometric series?

$$2+\left(-\frac{1}{2}\right)+\left(\frac{1}{8}\right)+\left(-\frac{1}{32}\right)+\cdots$$

A) $1\frac{3}{8}$

B) $1\frac{2}{5}$

C) $1\frac{1}{2}$

D) $1\frac{3}{5}$

E) $1\frac{5}{8}$

34) If $f(x)$ is a linear function and the slope of $y = f(x)$ is $\frac{1}{2}$, what is the slope of $y = f^{-1}(x)$?

A) -2 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 2 E) None of these

35) When a right triangle of area 3 square units is rotated 360 º about its shortest leg, the solid that results has a volume of 30 cubic units. What is the volume, in cubic units, of the solid that results when the same right triangle is rotated about its longer leg?

A) 0.99

B) 7.90

C) 8.88

D) 31.42

E) 41.12

36) How many integers are in the solution set of $|4x+3| < 8$?

A) Zero

B) Two

C) Three

D) Four

E) Infinitely many