

Algebra I/Integrated Math I 2011

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at University of Southern Indiana

ICTM Website

<http://www.indianamath.org/>

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Next year's math contest date: April 28, 2012

1. Factor: $x^2 + 15x - 54$
- A) $(x + 9)(x - 6)$ B) $(x - 9)(x + 6)$ C) $(x - 18)(x + 3)$
 D) $(x + 18)(x - 3)$ E) $(x + 27)(x - 2)$
2. Find the product: $(x + 3)(x - 3)^2$
- A) $x^3 - 27$ B) $x^3 + 27$ C) $x^3 - 3x^2 - 9x + 27$
 D) $x^3 + 3x^2 - 9x + 27$ E) $x^3 + 3x^2 - 9x - 27$
3. Simplify: $y(2x - y + 3) - 5(x^2 + xy - 2x) - x(3x - 7y + 1)$
- A) $14xy - 8x^2 - y^2 + 9x + 3y$
 B) $4xy - 8x^2 - y^2 - 11x + 3y$
 C) $-10xy - 8x^2 - y^2 + 9x + 3y$
 D) $4xy - 8x^2 - y^2 + 11x + 3y$
 E) $4xy - 8x^2 - y^2 + 9x + 3y$
4. Simplify: $-2(3a^3 + a^2) - 5(2a^3 - a^2 + a)$
- A) $-4a^3 + 3a^2 - 5a$ B) $-16a^3 - 3a^2 + 5a$ C) $-16a^3 + 3a^2 + 5a$
 D) $-16a^3 + 3a^2 - 5a$ E) $-16a^3 + 7a^2 - 5a$
5. Find the distance between the points $(5, 8)$ and $(-1, 5)$.
- A) 3 B) $3\sqrt{5}$ C) $\sqrt{-45}$ D) $\sqrt{153}$ E) $\sqrt{185}$
6. Factor: $mpx + mqx + npx + nqx$
- A) $(p + q)(m + n)$ B) $x(p + q)^2(m + n)$ C) $x(p + q)(m + n)^2$
 D) $x(p + q)^2(m + n)^2$ E) $x(p + q)(m + n)$

7. The proper dose of a children's antibiotic is 0.025 grams per kilogram of body mass. What is the proper dose for a child with a body mass of 45 kg?

A) 0.01125 g B) 1.125 g C) 18 g D) 1800 g E) none of these

8. Which of the following equations represents a line perpendicular to the line $14x - 5y = 60$?

A) $4x + 5y = -60$ B) $5x + 14y = 280$ C) $5x - 14y = 84$

D) $14x + 5y = -60$ E) $14x - 5y = 280$

9. An isosceles triangle is such that two of its sides measure 7.0 ft and the other side measures 8.0 ft. Find the height of the triangle to the nearest tenth of a foot.

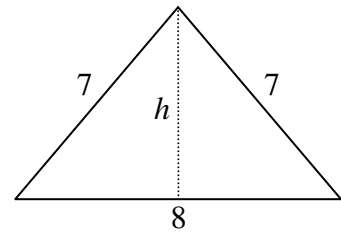
A) 1.0 ft

B) 3.9 ft

C) 5.7 ft

D) 8.1 ft

E) 10.6 ft



10. Solve the equation: $56 = (2t + 1)^2 + (2t + 1)$

A) $\{8, 7\}$ B) $\{-8, 7\}$ C) $\{8, -7\}$ D) $\{-4, \frac{7}{2}\}$ E) $\{3, -\frac{9}{2}\}$

11. Solve $t = qx^2$ for x for all $t > 0, q > 0$.

A) $x = \pm \frac{\sqrt{tq}}{q}$ B) $x = \pm \frac{\sqrt{tq}}{t}$ C) $x = \pm q\sqrt{tq}$ D) $x = \pm q\sqrt{t}$ E) none of these

12. Simplify:

$$\frac{6 + 3^2 + 12 + 3 - 9}{2(6 - 9) + 3^0 - 9 + (-3)}$$

A) -10 B) -5 C) $-\frac{10}{3}$ D) $\frac{13}{27}$ E) $\frac{1}{2}$

13. Solve the equation: $5 - 2(3x - 4) - 3^2 = 2x - 3(8 - 2x)$

- A) $\left\{\frac{1}{2}\right\}$ B) $\{2\}$ C) $\left\{\frac{23}{7}\right\}$ D) $\{6\}$ E) $\{15\}$

14. Solve the equation:

$$\frac{2 - 4(3x + 5)}{7} + 4 = \frac{2x}{3}$$

- A) $\{-1\}$ B) $\left\{-\frac{21}{35}\right\}$ C) $\left\{\frac{3}{5}\right\}$ D) $\left\{\frac{27}{16}\right\}$ E) $\{3\}$

15. Solve the inequality: $-4 \leq 6 - \frac{2}{3}x < 8$

- A) $(-\infty, -3) \cup [15, \infty)$ B) $(-\infty, -9) \cup [9, \infty)$ C) $(-9, 9)$
 D) $(-3, 15)$ E) $\left(-\frac{4}{3}, \frac{20}{3}\right]$

16. Plumbing Company A charges \$55 per hour for the first 2 hours and \$45 per hour beyond 2 hours. Company B charges \$60 per hour for the first 3 hours and \$40 per hour beyond 3 hours. For what number of hours is it cheaper to use Company A?

- A) more than 8 B) less than 8 C) more than 14 D) less than 14 E) none of these

17. Solve the inequality: $4|6 - 3x| \geq 48$

- A) $(-\infty, -2] \cup [6, \infty)$ B) $(-\infty, 6]$ C) $[-2, 6]$
 D) $[2, \infty)$ E) $(-\infty, -6] \cup [2, \infty)$

18. What is the value of the expression if $x = -2$?

$$-(-x - x^x)^{-x}$$

- A) -36 B) -4 C) $-\frac{49}{16}$ D) $\frac{9}{16}$ E) 4

19. Solve the equation: $-\frac{1}{3}(-4 + 3x) = \frac{1}{2}(2x - 3)$
- A) $\frac{17}{12}$ B) $\frac{17}{6}$ C) 9
 D) infinite number of solutions E) no solution
20. Find the product: $(x + 1)^2[x^2 - 2(x + 2)]$
- A) $x^4 + x^2 + 6x + 4$ B) $x^4 - 2x^2 - 3x^2 - 2x - 4$ C) $x^4 - 2x^2 + 5x^2 - 2x + 4$
 D) $x^4 - 7x^2 - 10x - 4$ E) none of these
21. Solve the given linear system for a and b . Then use those values to find c , where $c = ab^2$
- $$\begin{cases} a + b = 4 \\ a - 2b = 7 \end{cases}$$
- A) $c = -25$ B) $c = -5$ C) $c = 5$ D) $c = 25$ E) none of these
22. If $a \oplus b = (a + b)^2 - ab$, find $x \oplus 1$.
- A) $x^2 + x + 1$ B) $x^2 - x + 1$ C) $x^2 + 1$ D) $x^2 - 1$ E) $x^2 + 2x + 1$
23. The school bookstore sells notebooks at a price of two for five dollars and a package of ink pens for two dollars. Eight items are sold for nineteen dollars. If N is the number of notebooks and P is the number of packages of pens, which system of equations describes this purchase?
- A) $\begin{cases} 2N + P = 8 \\ 2.5N + 2P = 19 \end{cases}$ B) $\begin{cases} N + P = 8 \\ 7(N + P) = 19 \end{cases}$ C) $\begin{cases} N + P = 8 \\ 5N + 2P = 19 \end{cases}$
 D) $\begin{cases} N + P = 8 \\ 2.5N + 2P = 19 \end{cases}$ E) none of these
24. If you are travelling at 85 ft/sec, approximately how fast are you going in miles per hour? (note: 5280 ft = 1 mile)
- A) 17 mph B) 39 mph C) 45 mph D) 58 mph E) 91 mph

25. If $f(x) = x^3 - x^2 + 4$, find $f(2\sqrt{3})$.
- A) $16\sqrt{3}$ B) $24\sqrt{3}$ C) $32\sqrt{3}$ D) $4 + 15\sqrt{3}$ E) $-8 + 24\sqrt{3}$
26. Solve the system of equations:
- $$\begin{cases} \frac{3}{5}x + \frac{4}{5}y = 5 \\ x - \frac{2}{3}y = \frac{19}{3} \end{cases}$$
- A) $(3, -5)$ B) $(5, 5)$ C) $\left(\frac{43}{7}, \frac{33}{7}\right)$ D) $(7, 1)$ E) $(9, 4)$
27. Nick and Rhonda are building a square fish pond in their backyard. They wish to put a 2-foot wide brick border around the outside so that the pond plus the border is in the shape of a square. If the total area of the border and the fish pond is 196 square feet, find the dimensions of one side of the fish pond.
- A) 6 feet B) 10 feet C) 12 feet D) 16 feet E) 18 feet
28. Solve the equation: $\sqrt{r^2 - 5r + 13} - 2 = r$
- A) $\{-1\}$ B) $\{1\}$ C) $\left\{\frac{11}{5}\right\}$ D) \emptyset E) none of these
29. Tickets for a firemen's ball are \$25 per person or \$40 per couple. Assume x represents the number of tickets for singles and y represents the number of tickets for couples. If the goal is to sell at least \$4800 worth of tickets, which inequality best models that goal?
- A) $25x + 40y > 4800$ B) $25x + 80y > 4800$ C) $25x + 40y \geq 4800$
 D) $25x + 80y \geq 4800$ E) none of these
30. Simplify: $\sqrt[3]{81a^3b^3} + \sqrt[3]{24a^3b^3}$
- A) $(3ab^2 + 2a^2)\sqrt[3]{3b^2}$ B) $\sqrt[6]{105a^9b^{10}}$ C) $3ab^2 + 2a^2$
 D) $a^3b^3\sqrt[3]{105b}$ E) $3b^2(\sqrt[3]{3ab^2} + \sqrt[3]{2a^2})$

31. Solve the equation and determine the sum of the solutions: $x^3 - x^2 = 4x - 4$
- A) -1 B) 0 C) 1 D) 4 E) none of these
32. Below is a stem and leaf plot. Which two values for this data are the same?
- | | |
|---|---------|
| 0 | 8 |
| 1 | 3 7 7 9 |
| 2 | 2 6 8 |
| 3 | 0 2 2 2 |
- A) mean and median B) mean and mode C) mean and range
- D) median and mode E) median and range
33. If $f(x) = x^2 - x + 3$, find $f(x - 2)$.
- A) $x^2 - x + 1$ B) $x^2 - x - 3$ C) $x^2 - 5x + 5$ D) $x^2 - 5x + 9$ E) $x^2 + 3x + 5$
34. What is the domain of the relation $x = y^2 - 5$?
- A) $\{x|x \neq -5\}$ B) $\{x|x \neq 5\}$ C) $\{x|x \geq -5\}$
- D) $\{x|-2 \leq x \leq 2\}$ E) $\{x|-5 \leq x \leq 5\}$

For the remainder of the test, assume all denominators to be non-zero.

35. Solve for x : $2xy + 5z = 3 - 4xz + 8xy$
- A) $x = \frac{5z - 3}{6y - 4z}$ B) $x = \frac{5z + 4xz - 3}{6y}$ C) $x = \frac{5z - 3}{2yz}$
- D) $x = \frac{5z - 3}{4yz - 2y}$ E) none of these

36. Calculate the slope of the line through points (a, h) and $(a + h, a)$, where $h \neq 0$.

- A) $-a$ B) $a - 1$ C) $\frac{h - a}{h}$ D) $\frac{a - h}{h}$ E) $\frac{h}{a - h}$

37. Write the rational expression in lowest terms:

$$\frac{6 + x - x^2}{x^2 + x - 12}$$

- A) $-\frac{1}{2}$ B) $-\frac{x + 2}{x + 4}$ C) $\frac{x + 2}{x + 4}$ D) $\frac{x + 3}{x + 6}$ E) $\frac{3 - x}{x + 6}$

38. Simplify:

$$\frac{24h^2k + 56hk^2 - 28hk}{16h^2k^2}$$

- A) $\frac{3}{2k} + \frac{7}{2h} - \frac{7}{4hk}$ B) $3k + 56h - 14hk$ C) $\frac{3k}{2} + \frac{7h}{2} - \frac{7hk}{4}$
 D) $\frac{3 + 56hk^2 - 28hk}{2k}$ E) none of these

39. Multiply and divide as indicated, then write the rational expression in lowest terms:

$$\frac{9x^2 - 16}{x^2 + x - 42} \times \frac{x^2 + 5x - 14}{3x^2 - 10x + 8} \div \frac{3x^2 + x - 4}{6 - x}$$

- A) $-(x + 1)$ B) $-1 + x$ C) $\frac{-1}{x - 1}$ D) $\frac{-1}{1 - x}$ E) none of these

40. Solve:

$$\frac{5a + 2}{2a} = \frac{18}{a + 4}$$

- A) $\left\{\frac{2}{31}, \frac{4}{35}\right\}$ B) $\left\{\frac{5}{4}, 2\right\}$ C) $\left\{\frac{1}{5}\right\}$ D) $\left\{\frac{2}{5}, 4\right\}$ E) $\left\{\frac{4}{5}, 2\right\}$