Algebra I/Integrated Math I

2009

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **University of Southern Indiana**

ICTM Website

http://www.indiana math.org/

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Next year's math contest date: April 24, 2010

1. Simplify: $-\left(5x^3-8x^2+x-6\right)+\left(3x^3-9x+2\right)-\left(-7x^2+4x\right)$

a) $-2x^3+15x^2-4x-4$

b) $-2x^3+15x^2-6x+8$

c) $-2x^3+15x^2-12x-4$

d) $-2x^3+15x^2-14x+8$

e) None of these

2. Simplify: $\left(2^{-3}+\left(\frac{1}{3}\right)^{-2}\right)^{-1}$

a) $-\frac{73}{8}$ b) $-\frac{37}{6}$ c) $-\frac{6}{37}$ d) $\frac{8}{73}$ e) $\frac{73}{9}$

3. Find the product of polynomials P and Q: $P(x)=\left(x-1\right)^2$ and $Q(x)=\left(x+3\right)$

a) x^3+x^2-4x+3

b) x^3+x^2-5x+3

c) x^3+x^2-7x-3

d) x^3+3x^2-x-3

e) $2x^2+4x-6$

4. Solve the equation: $-3[6-2(1+5x-7)]=8x-2(-5+3x)$

a) $\left\{-\frac{4}{3}\right\}$ b) $\left\{\frac{31}{37}\right\}$ c) $\left\{\frac{43}{31}\right\}$ d) $\left\{\frac{16}{7}\right\}$ e) None of these

5. Solve the equation: $\sqrt{x^2+3}=6+x$

a) $\left\{-\frac{11}{4}\right\}$ b) $\left\{-\frac{11}{4},\frac{11}{4}\right\}$ c) $\left\{-\frac{13}{4},\frac{13}{4}\right\}$ d) $\left\{\frac{13}{4}\right\}$ e) None of these

6. Solve the inequality: $-1<5-\frac{2}{3}x\le 6$

a) $[-\frac{3}{2},9)$ b) $[-\frac{2}{3},4)$ c) $[-9,9)$ d) $(-\infty,-\frac{3}{2}]\cup(9,\infty)$ e) None of these

7. Solve the equation: $3(x-5)^2 = 54$

a) $\{-\sqrt{23}, \sqrt{23}\}$

b) $\{5-3\sqrt{2},\ 5+3\sqrt{2}\}$

c) $\{5-9\sqrt{2},\ 5+9\sqrt{2}\}$

d) $\{-4, 14\}$

e) $\varnothing$

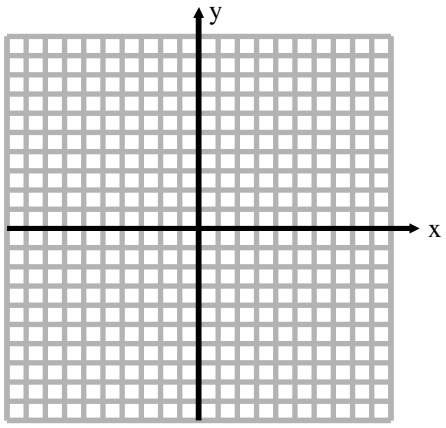
8. Solve the given equation and determine the sum of the two solutions: $3x^2 + 7x - 40 = 0$

a) $-\frac{14}{3}$ b) $-\frac{7}{3}$ c) $-\frac{2}{3}$ d) $\frac{2}{3}$ e) $\frac{7}{3}$

9. In the Kentucky Derby, Prancer ran 6 furlongs in 1 minute and 6 seconds. What is Prancer's rate in miles per hour to the nearest tenth of a mile? (Note: 1 mile equals 8 furlongs)

a) 28.1 b) 30.0 c) 36.1 d) 40.9 e) 48.0

10. Line l contains the point, $\left(\frac{1}{3}, -\frac{2}{5}\right)$, and is perpendicular to the line, $x = -\frac{3}{2}y - 6$. In which quadrants does the graph of line l lie?



a) I, II, and III

b) I, II, and IV

c) I, III, and IV

d) I and III

e) II and IV

11. A company produces CDs of concerts. The equipment costs the company \$13440. It also costs the company \$20 for each CD produced. They charge \$37.50 for each CD sold. How many CDs need to be sold to make a profit?

a) 359 b) 672 c) 717 d) 750 e) 769

12. Factor the expression: $81-(5+x)^2$

a) $(9+x)(9-x)$

b) $(14+x)(14-x)$

c) $(14+x)(4-x)$

d) $(4+x)(14+x)$

e) None of these

13. Write a polynomial function, $A(x)$, that models the shaded area.

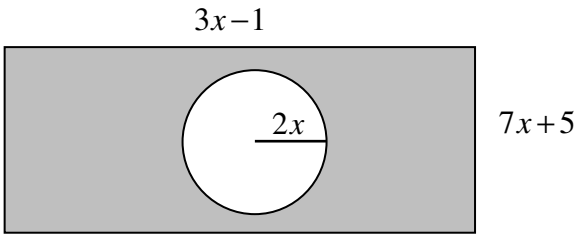
a) $A(x)=(21-4\pi)x^2+8x-5$

b) $A(x)=(21-2\pi)x^2+8x-5$

c) $A(x)=-4x^2+10x+4$

d) $A(x)=23x^2+8x-5$

e) None of these



14. The expression $\dfrac{\sqrt{2}}{\sqrt{2}-1}$ is equivalent to which of the following:

a) $2-\sqrt{2}$ b) -1 c) $2+\sqrt{2}$ d) -2 e) $3-2\sqrt{2}$

15. Emily purchased stock on Company A at \$14.75 per share and purchased stock in Company B at \$37.50 per share. If Emily purchased a total of 650 shares at a cost of \$14,137.50, how many shares of company B stock did she purchase?

a) 150 b) 200 c) 300 d) 325 e) 450

16. Solve the equation: $\dfrac{2x+1}{x}=\dfrac{3}{x}$

a) $\{0\}$ b) $\{1\}$ c) $\{0,1\}$ d) $\{x \mid x \text{ is a real number}\}$ e) $\varnothing$

17. In the box on the right, which ones represent functions?

a) I and II
b) II and III
c) I, II, and III
d) I, II, and IV
e) I, III, and IV

I.	$\{(2,3),(-2,3),(-3,2)\}$
II.	$x = y^2$
III.	$y = x^2$
IV.	$\begin{cases} \text{Zach} \to \text{boy} \\ \text{Tim} \to \text{boy} \\ \text{Andrea} \to \text{girl} \\ \text{Nicole} \to \text{girl} \end{cases}$

18. Given $f(x) = -2x^2 - x$, find $f(x+a)$.

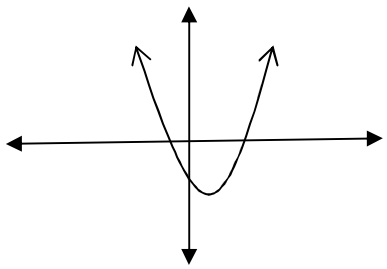
a) $-2x^2 - x + a$
b) $-2x^2 - 2a^2 - x - a$
c) $-2x^2 - 4ax - 2a^2 - x$
d) $-2x^2 - 4ax - 2a^2 - x - a$
e) $-2x^2 - 4ax - 2a^2 - x + a$

19. Solve the equation: $x^2 + 2x - 7 = 0$

a) $\{-1 \pm 2\sqrt{2}\}$ b) $\{-2 \pm \sqrt{2}\}$ c) $\left\{\frac{-2 \pm \sqrt{2}}{2}\right\}$ d) $\left\{\frac{-2 \pm \sqrt{-24}}{2}\right\}$ e) $\left\{\frac{-2 \pm \sqrt{24}}{2}\right\}$

20. The range of the graphed function with vertex (1, -3) is:

a) $(-\infty, \infty)$
b) $[1, \infty)$
c) $(-3, \infty)$
d) $(1, -3)$
e) $[-3, \infty)$



21. Skim milk is 0.3% fat. Whole milk is 3.3% fat. How many quarts of skim milk and whole milk must be mixed together to make 27 quarts of milk that is 2% fat?

a) 11.3 quarts of skim and 15.7 quarts of whole
b) 11.4 quarts of skim and 15.6 quarts of whole
c) 11.5 quarts of skim and 15.5 quarts of whole
d) 11.6 quarts of skim and 15.4 quarts of whole
e) 11.7 quarts of skim and 15.3 quarts of whole

22. Solve the equation: $\frac{x+1}{4}-\frac{2-x}{3}=\frac{1}{6}$

a) $\left\{\frac{6}{7}\right\}$ b) $\{1\}$ c) $\left\{\frac{7}{4}\right\}$ d) $\left\{\frac{9}{2}\right\}$ e) $\{7\}$

23. Find the range of $f(x)=x^2+x-1$, if the domain is $\{-2,-1,0,1,2\}$.

a) $\{-1,1,5\}$
b) $\{-1,1,0,3,7\}$
c) $\{1,3,7\}$
d) $\{-7,-3,-1,1,5\}$
e) $\{-7,-1,1,3,5\}$

24. Solve the inequality: $3|7-2x|\geq 45$

a) $[-4,11]$ b) $[-11,4]$ c) $(-\infty,-4]\cap[11,\infty)$ d) $(-\infty,-4]\cup[11,\infty)$ e) None of these

25. The linear function f is such that $f(-1)=7$ and $f(2)=1$. Find $f(4)$.

a) -5 b) -3 c) -1 d) 1 e) $\frac{11}{3}$

26. Determine the domain of the function: $f(x)=\frac{3x^2+5x+2}{x^2-1}$

a) $\{x \mid x \text{ is a real number}, x\neq 1\}$
b) $\{x \mid x \text{ is a real number}, x\neq -1, x\neq 1\}$
c) $\{x \mid x \text{ is a real number}, x\neq -1, x\neq -\frac{2}{3}, x\neq 1\}$
d) $\{x \mid x \text{ is a real number}, x\neq -1, x\neq \frac{2}{3}, x\neq 1\}$
e) $\{x \mid x \text{ is a real number}\}$

27. Simplify: $\frac{|8+(-12)|-2(-3)^2}{2^0+\sqrt{12^2+16^2}}$

a) $-\frac{2}{3}$ b) $-\frac{14}{29}$ c) $-\frac{2}{21}$ d) 6 e) 22

28. Which number is $\frac{3}{5}$ of the way from -1.5 to 2.7?

a) -0.78 b) 1.02 c) 1.78 d) 2.02 e) 2.52

29. The following expression represents the number of seconds a gerbil participated in an experiment: $\frac{(4\times10^{8})(4.6656\times10^{-2})}{8\times10^{-1}}$. How many days is this?

a) 2.7 b) 27 c) 270 d) 2700 e) None of these

30. Solve the system of equations, $\begin{cases} 3y = x - 2y - 1 \\ 3x = 2x - y - 1 \end{cases}$, for x and y. The sum of x and y is:

a) -1 b) $-\frac{1}{2}$ c) $-\frac{21}{65}$ d) 0 e) $\frac{29}{11}$

31. If $(-3,6)$, $\left(1,\frac{9}{2}\right)$, and $(8,a)$ are all points on the same line, find the value of a.

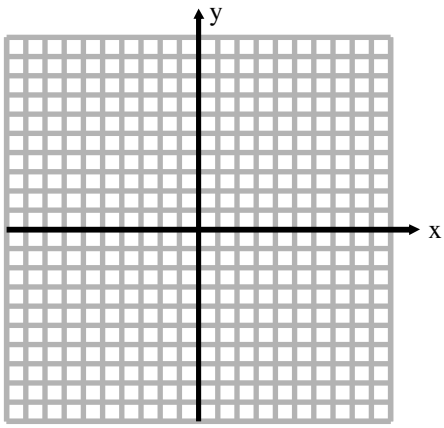
a) $-\frac{70}{3}$ b) $-\frac{25}{3}$ c) $\frac{15}{8}$ d) 1.9 e) 2

32. Which equation below represents the line that passes through the midpoint of the segment connecting the points $(-1,-5)$ and $(13,-1)$ and is also parallel to the line $3x-6y=10$?

a) $x+2y=0$
b) $x-2y=6$
c) $x-2y=12$
d) $2x+y=9$
e) $2x-y=15$

33. What is the shape of the region bounded by the following system of inequalities:

$$\begin{cases} x+2y \le 16 \\ x-y \ge -2 \\ 1 \le x \le 4 \\ y \ge -1 \end{cases}$$



a) an acute triangle
b) a right triangle
c) an obtuse triangle
d) a trapezoid
e) a pentagon

34. The time it takes to complete a construction job varies inversely as the number of people working on the job. If it takes a crew of 21 workers 3 weeks to complete a construction job, how many construction workers would be required to complete the job in only 9 days? Assume a 5-day work week for the construction workers.

a) 8 b) 9 c) 11 d) 27 e) 35

For the remainder of the test, assume all denominators to be non-zero.

35. What polynomial, when divided by $2x-1$ has a quotient of $5x+3$?

a) $10x^2+x-3$ b) $\frac{5}{2}x-3$ c) $7x+2$ d) $7x^2+x+3$ e) None of these

36. Find the least common denominator for the given fractions: $\frac{17}{3ab^2}$ and $\frac{13}{9ab}$

a) $3ab$ b) $3ab^2$ c) $9ab$ d) $9ab^2$ e) $27a^2b^3$

37. The expression $\frac{x^2-4}{x^2+7x+10}\cdot\frac{x^2+x-20}{x^2-5x+4}$ is equivalent to which of the following:

a) $\frac{x-2}{x-1}$ b) $\frac{x-2}{x+1}$ c) $\frac{2}{35x}$ d) $\frac{x^2-24}{2x^2+14}$ e) $\frac{x^4+x^3-24x^2-4x-80}{x^4+2x^3-21x^2-22x+40}$

38. Solve for P: $A-P=Prt$

a) $P=\frac{A-P}{rt}$ b) $P=\frac{A}{rt}$ c) $P=\frac{A}{rt+1}$ d) $P=\frac{A}{rt-1}$ e) $P=\frac{A}{1-rt}$

39. The expression $(x-3y)^{-1}-(x+3y)^{-1}$ is equivalent to which of the following:

a) 0 b) $6xy$ c) $\frac{6}{y}$ d) $\frac{6y}{x^2-9y^2}$ e) $\frac{2}{x^2-6xy-9y^2}$

40. The expression $1-\frac{1}{x-1}-\frac{2}{x^2-1}$ is equivalent to which of the following:

a) $\frac{x-4}{x+1}$ b) $x+4$ c) x^2-x-4 d) $\frac{-x^2-x-4}{(x+1)(x-1)}$ e) $\frac{x^2-x-4}{(x+1)(x-1)}$